SAT Solvers

Ranjit Jhala, UC San Diego

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Decision Procedures

We will look very closely at the following

1. Propositional Logic
2. Theory of *Equality*
3. Theory of *Uninterpreted Functions*
4. Theory of *Difference-Bounded Arithmetic*

Decision Problem: Satisfaction

- Does `eval s p` return `True` for **some** assignment `s` ?
- *"Can we assign the variables to make the formula true"* ?

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3. Theory of *Uninterpreted Functions*
4. Theory of *Difference-Bounded Arithmetic*

Why?

- Representative
- Have *"efficient"* algorithms

Decision Procedures

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3. Theory of *Uninterpreted Functions*
4. Theory of *Difference-Bounded Arithmetic*

Plan

- First in **isolation**
- Then in **combination**
- Very slick SW-Eng, based on logic

Decision Procedures: Propositional Logic

Popularly called **SAT Solvers**

Decision Procedures: Propositional Logic

Basics

- Propositional Logic 101
- Conjunctive Normal Form
- Resolution

Algorithms

- Resolution
- Backtracking Search
- Boolean Constraint Propagation
- Conflict Driven Learning & Backjumping

Decision Procedures: Propositional Logic

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Propositional Logic 101

Propositional Variables

```
data PVar
```

Propositional Formulas

```
data Formula = Prop PVar
             | Not  Formula
             | Formula `And` Formula
             | Formula `Or`  Formula
```

Decision Procedures: Propositional Logic

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Conjunctive Normal Form

Restricted representation of `Formula`

Literals: Variables or Negated Variables

```
data Literal    = Pos PVar | Neg PVar
```

Clauses: Disjunctions (Or) of Literals

```
data Clauses    = [Literal]
```

CNF Formulas: Conjunctions (And) of Clauses

```
data CnfFormula = [Clauses]
```

Conjunctive Normal Form: Example

Consider a Formula

$(x_1 \vee x_2) \wedge (\neg x_1 \vee x_3) \wedge \neg x_3$

Represented as a Formula

```
      (Prop 1          `Or` Prop 2)
`And` (Not (Prop 1) `Or` Prop 3)
`And` (Not (Prop 3)                )
```

Represented as a CnfFormula

```
    [ [Pos 1 , Pos 2]
    , [Neg 1 , Pos 3]
    , [Neg 3       ] ]
```

Conjunctive Normal Form Conversion

Theorem There is a *poly-time* function

```
toCNF :: Formula -> CnfFormula
toCNF = error "Exercise For The Reader"
```

Such that any `f` is satisfiable *iff* `(toCNF f)` is satisfiable.

- `toCNF` adds **new variables** for sub-formulas
- otherwise, an **exponential blowup** in `CnfFormula` size

Conjunctive Normal Form Conversion

Theorem There is a *poly-time* function

```
toCNF :: Formula -> CnfFormula
toCNF = error "Exercise For The Reader"
```

Such that any `f` is satisfiable *iff* `(toCNF f)` is satisfiable.

Henceforth Only consider formulas in Conjunctive Normal Form Formulas

Decision Procedures: Propositional Logic

Basics

- Propositional Logic 101
- Conjunctive Normal Form

Algorithms

- **Resolution**
- Backtracking Search
- Boolean Constraint Propagation
- Conflict Driven Learning & Backjumping

Properties of CNF

Pure Variable

- One which appears only $+ve$ or $-ve$ in a `CnfFormula`

Empty Clause

- **If** a `CnfFormula` has *some* `Clause` without `Literals`
- **Then** the `CnfFormula` is **UNSAT**

Trivial Formula

- **If** a `CnfFormula` has *no* `Clause`
- **Or** every variable is *pure*
- **Then** the `CnfFormula` is **SAT**

Goal

Determine satisfaction by **reducing** `CnfFormula` to one of

- Empty Clause (ie *UNSAT*), or
- Trivial Formula (ie *SAT*).

Reducing Formulas By Resolution

("Reduce" is, perhaps, not the best word...)

Resolution: For any A, B and variable x, the formula

$$(A \vee x) \wedge (B \vee \neg x)$$

is *equivalent to* the formula

$$(A \vee B)$$

- The variable x is called a **pivot** variable

General Resolution

Resolution: For any A_i, B_j and variable x, the formula

$$\bigwedge_i (A_i \vee x) \wedge \bigwedge_j (B_j \vee \neg x)$$

is *equivalent to* the formula

$$\bigwedge_{i,j} (A_i \vee B_j)$$

- Pivot variable x is **eliminated** by resolution

Davis-Putnam Algorithm: Example 1

Input Formula

- $(x_1 \vee x_2 \vee x_3) \wedge (x_2 \vee \neg x_3 \vee x_5) \wedge (\neg x_2 \vee x_4))$

Pivot on x_2

- $(x_1 \vee x_3 \vee x_4) \wedge (\neg x_3 \vee x_5 \vee x_4)$

Pivot on x_3

- $(x_1 \vee x_4 \vee x_5)$

All variables are *pure* . . . hence, **SAT**

Davis-Putnam Algorithm: Example 2

Input Formula

- $(x_1 \vee x_2) \wedge (x_1 \vee \neg x_2) \wedge (\neg x_1 \vee x_3) \wedge (\neg x_1 \vee \neg x_3)$

Pivot on x_2

- $(x_1) \wedge (\neg x_1 \vee x_3) \wedge (\neg x_1 \vee \neg x_3)$

Pivot on x_3

- $(x_1) \wedge (\neg x_1)$

Pivot on x_1

- $()$

Empty clause ... hence, **UNSAT**

Davis-Putnam Algorithm

Algorithm

1. Select **pivot** and perform **resolution**
2. Repeat until **SAT** or **UNSAT**

Issues?

- Space blowup (formula size blows up on resolution)

Decision Procedures: Propositional Logic

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Algorithms

- Resolution
- **Backtracking Search**
- Boolean Constraint Propagation
- Conflict Driven Learning & Backjumping

Decision Tree: Describes Space of All Assignments

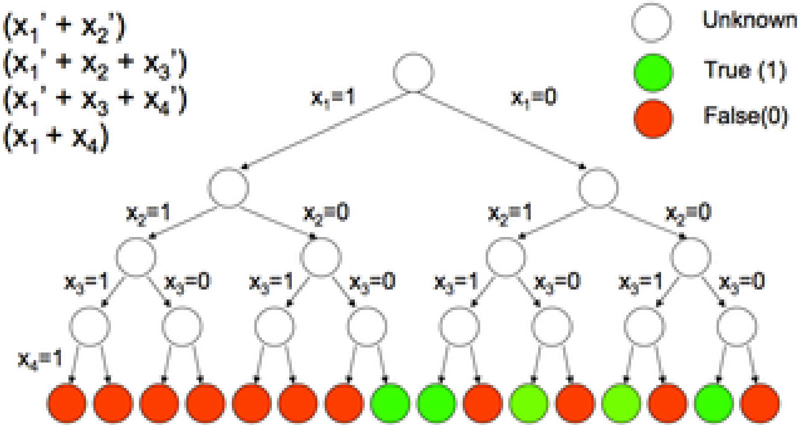

Figure: SAT Decision Tree (Courtesy: Lintao Zhang)

Decision Tree: SAT via Depth First Search

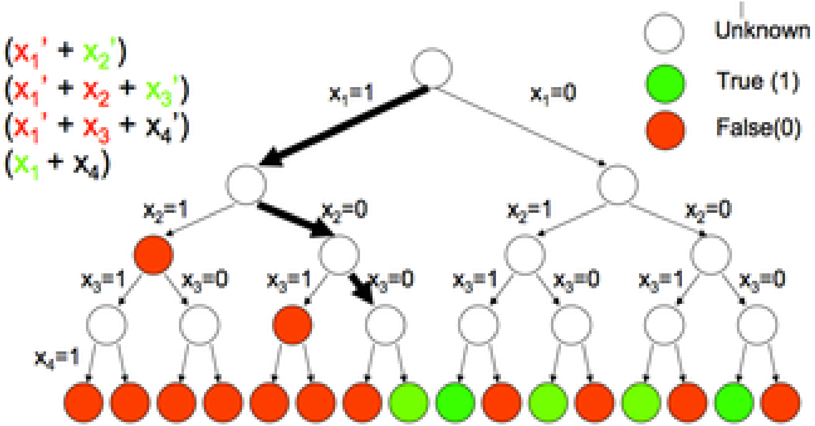

Figure: DFS On Decision Tree (Courtesy: Lintao Zhang)

Backtracking Search

Don't build *whole* tree, but lazily search solutions

- **Choose** a variable x, set to `True`
- **Remove** constraints where x appears
- **Recurse** on remaining constraints
- **Backtrack** if a contradiction is found

Backtracking Search (1/21)

(a' + b + c)
(a + c + d)
(a + c + d')
(a + c' + d)
(a + c' + d')
(b' + c' + d)
(a' + b + c')
(a' + b' + c)

Figure: Basic DLL (Courtesy: Lintao Zhang)

Backtracking Search (2/21)

(a' + b + c)
(a + c + d)
(a + c + d')
(a + c' + d)
(a + c' + d')
(b' + c' + d)
(a' + b + c')
(a' + b' + c)

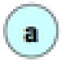

Figure: Basic DLL (Courtesy: Lintao Zhang)

Backtracking Search (3/21)

(a' + b + c)
(a + c + d)
(a + c + d')
(a + c' + d)
(a + c' + d')
(b' + c' + d)
(a' + b + c')
(a' + b' + c)

Figure: Basic DLL (Courtesy: Lintao Zhang)

Backtracking Search (4/21)

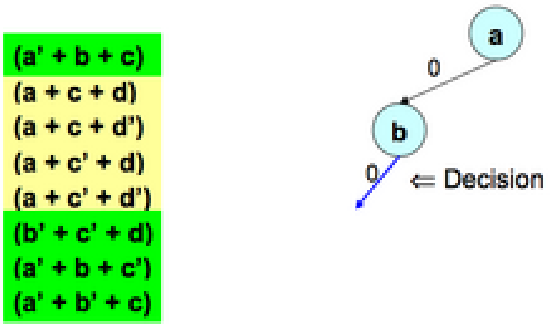

Figure: Basic DLL (Courtesy: Lintao Zhang)

Backtracking Search (5/21)

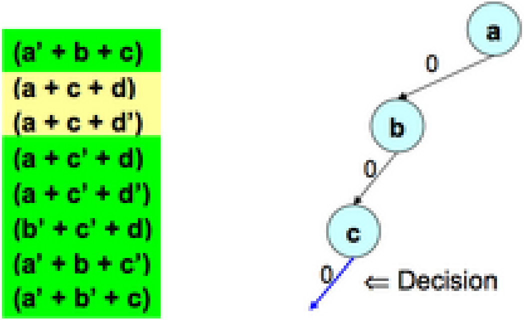

Figure: Basic DLL (Courtesy: Lintao Zhang)

Backtracking Search (6/21)

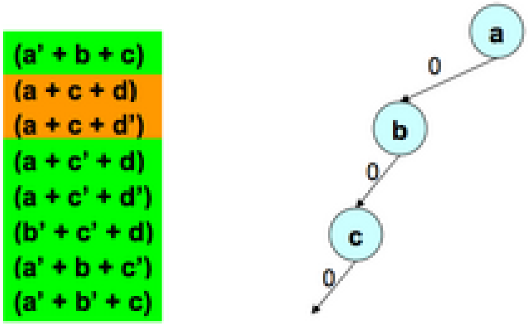

Figure: Basic DLL (Courtesy: Lintao Zhang)

Backtracking Search (7/21)

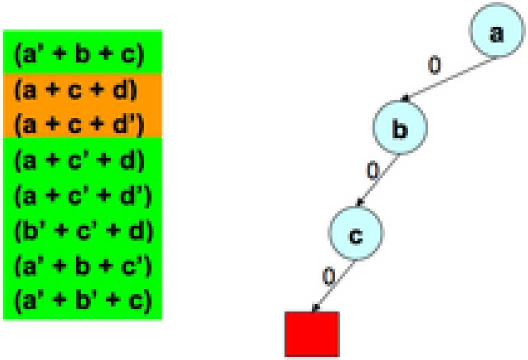

Figure: Basic DLL (Courtesy: Lintao Zhang)

Backtracking Search (8/21)

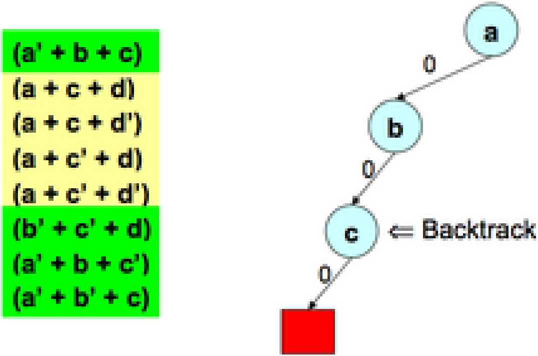

Figure: Basic DLL (Courtesy: Lintao Zhang)

Backtracking Search (9/21)

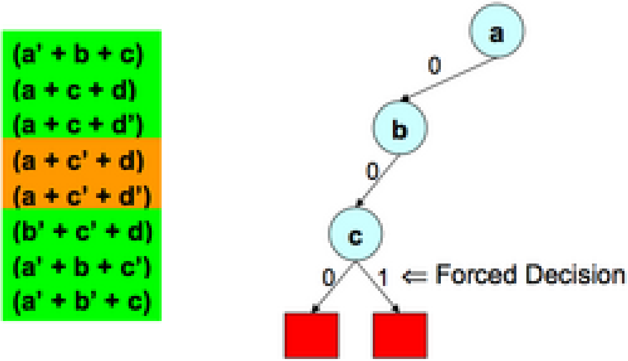

Figure: Basic DLL (Courtesy: Lintao Zhang)

Backtracking Search (10/21)

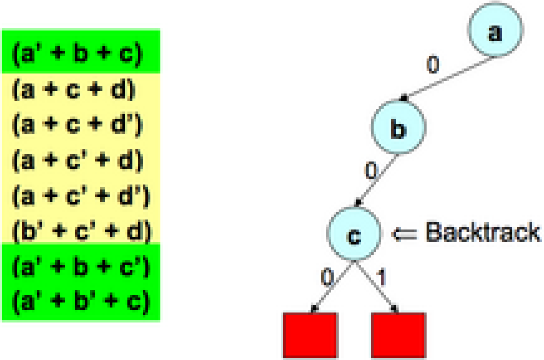

Figure: Basic DLL (Courtesy: Lintao Zhang)

Backtracking Search (11/21)

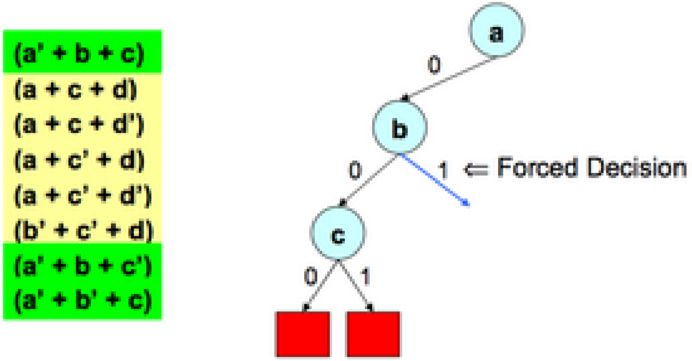

Figure: Basic DLL (Courtesy: Lintao Zhang)

Backtracking Search (12/21)

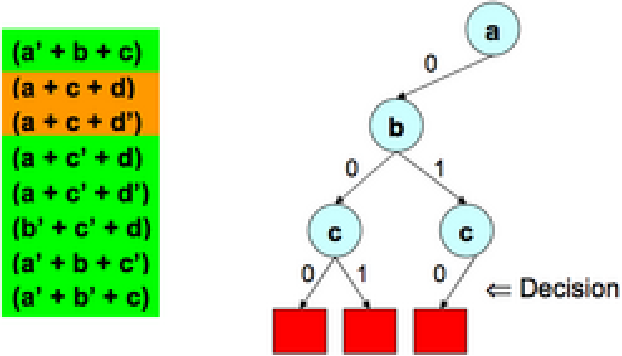

Figure: Basic DLL (Courtesy: Lintao Zhang)

Backtracking Search (13/21)

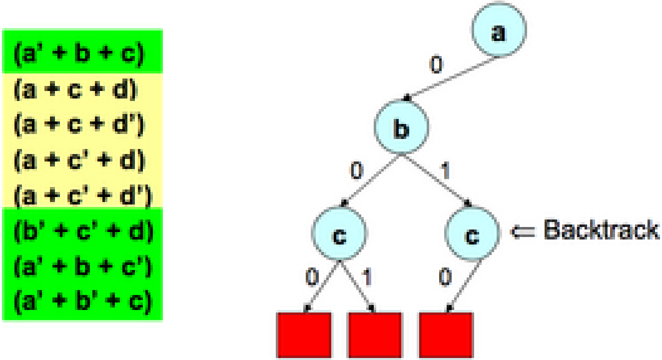

Figure: Basic DLL (Courtesy: Lintao Zhang)

Backtracking Search (14/21)

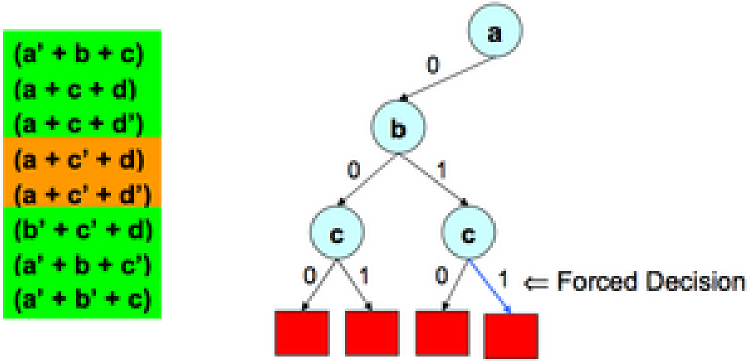

Figure: Basic DLL (Courtesy: Lintao Zhang)

Backtracking Search (15/21)

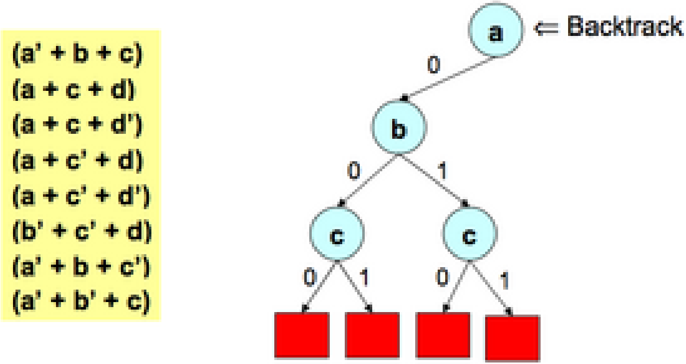

Figure: Basic DLL (Courtesy: Lintao Zhang)

Backtracking Search (16/21)

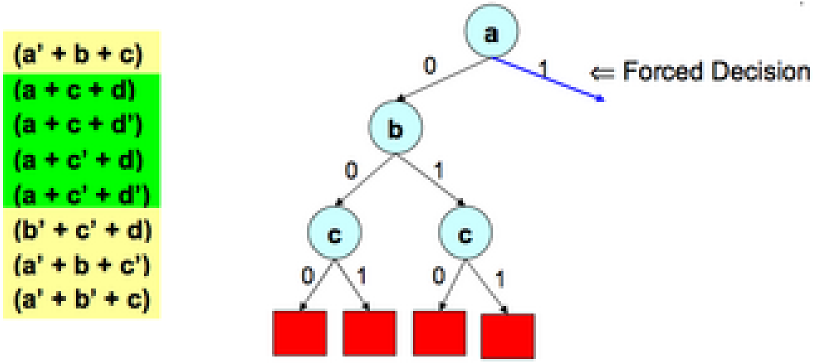

Figure: Basic DLL (Courtesy: Lintao Zhang)

Backtracking Search (17/21)

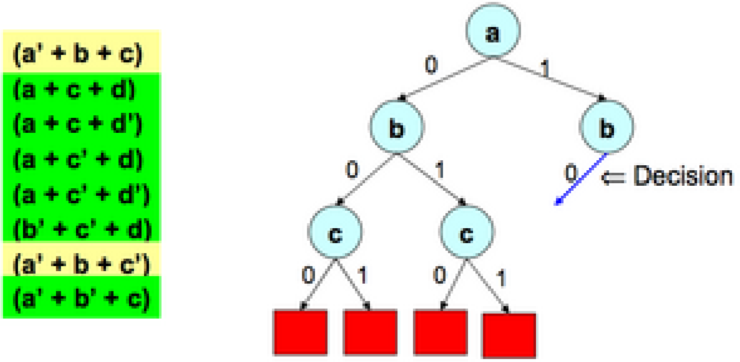

Figure: Basic DLL (Courtesy: Lintao Zhang)

Backtracking Search (18/21)

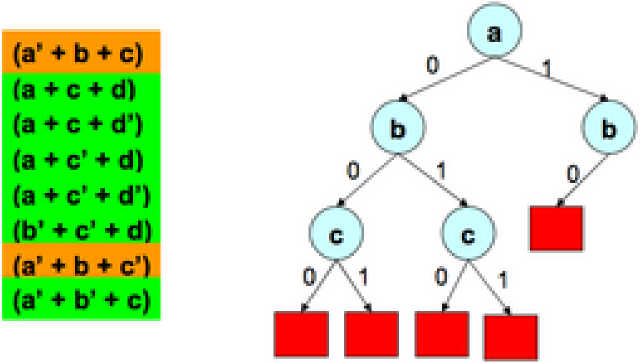

Figure: Basic DLL (Courtesy: Lintao Zhang)

Backtracking Search (19/21)

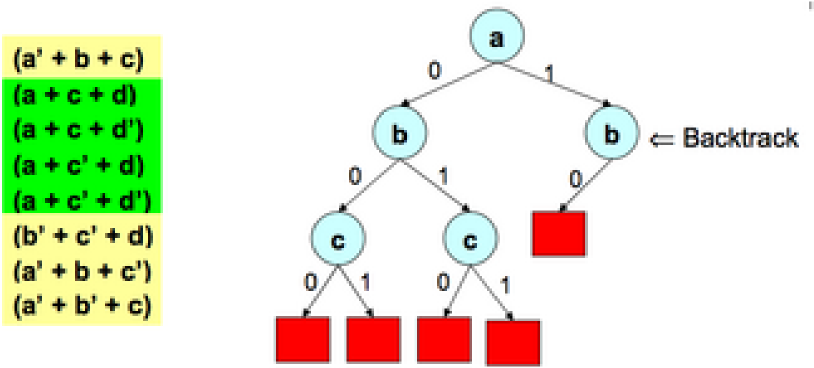

Figure: Basic DLL (Courtesy: Lintao Zhang)

Backtracking Search (20/21)

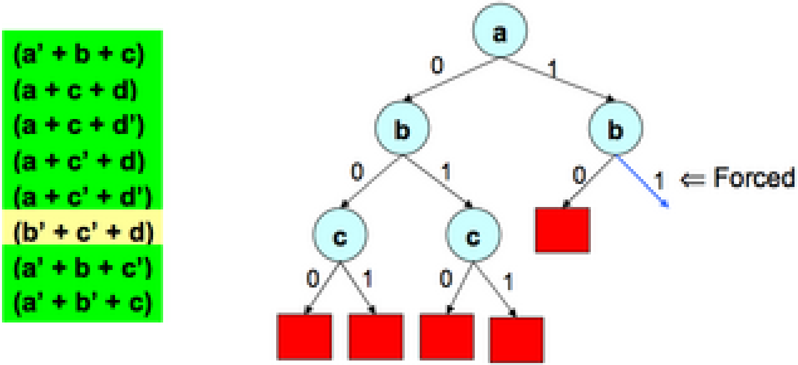

Figure: Basic DLL (Courtesy: Lintao Zhang)

Backtracking Search (21/21)

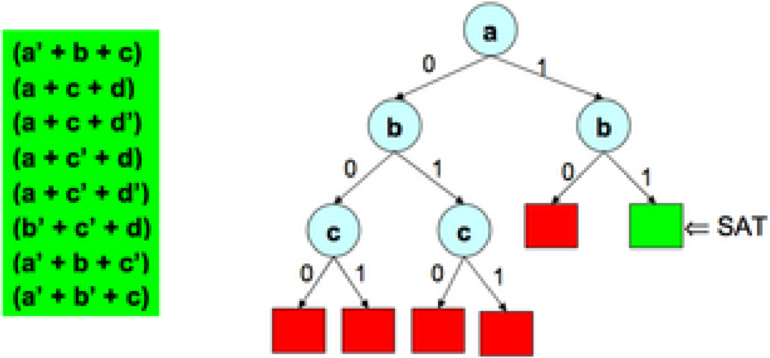

Figure: Basic DLL (Courtesy: Lintao Zhang)

Backtracking Search

Don't build *whole* tree, but lazily search solutions

- **Choose** a variable x, set to `True`
- **Remove** constraints where x appears
- **Recurse** on remaining constraints
- **Backtrack** if a contradiction is found

(*whew!*)

- DFS avoids *space* blowup (only need to save stack) ...
- ... but not time (natch)

Decision Procedures: Propositional Logic

Basics

- Propositional Logic 101
- Conjunctive Normal Form

Algorithms

- Resolution
- Backtracking Search
- **Boolean Constraint Propagation**
- Conflict Driven Learning & Backjumping

Boolean Constraint Propagation

Often, we don't really have a choice. . .

Boolean Constraint Propagation

Unit Clause Rule

- **If** an (unsatisfied) `Clause` has **one** unassigned `Literal`
- **Then** that `Literal` **must** be `True` in any SAT assignment

Example

- **Formula** $(x_1 \vee \neg x_2 \vee x_3) \wedge (x_2 \vee \neg x_3) \wedge (\neg x_1 \vee \neg x_3)$
- **Assignment** $x_1 = T, x_2 = T$
- The **last** clause is a unit clause
- Any SAT assigment **must** set $\neg x_3 = T$ (i.e. $x_3 = F$)

Boolean Constraint Propagation

Unit Clause Rule

- **If** an (unsatisfied) `Clause` has **one** unassigned `Literal`
- **Then** that `Literal` **must** be `True` in any SAT assignment

BCP or Unit Propagation

- **Repeat** applying *unit clause rule*
- **Until** no unit clause remains.

Boolean Constraint Propagation: Example

Revisit Example With BCP

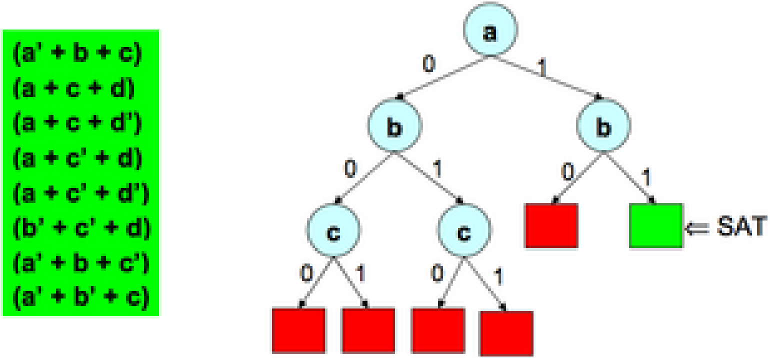

Figure: Boolean Constraint Propagation (Courtesy: Lintao Zhang)

Boolean Constraint Propagation

DPLL = Backtracking Search + BCP

- Backtracking: Avoids space blowup
- BCP: Avoid doing obvious work
- Still repeatedly explore all choices (e.g. whole left subtree)

Wanted

- Means to *learn* to repeat *dead ends*
- Key to scaling to practical problems

Decision Procedures: Propositional Logic

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- **Conflict Driven Learning & Backjumping**

Conflict Driven Learning

Key Insight

- On finding conflict, don't (just) backtrack
- **Learn new clause** to prevent same conflict in future

Major breakthrough

- J. P. Marques-Silva and K. A. Sakallah, "GRASP – A New Search Algorithm for Satisfiability," Proc. ICCAD 1996.
- R. J. Bayardo Jr. and R. C. Schrag "Using CSP look-back techniques to solve real world SAT instances." Proc. AAAI, 1997

Conflict Driven Learning

- Resolve on conflict variable to **learn** new **conflict** clause
- **Add** clause to set of clauses
- **Backjump** using conflict clause

Conflict Driven Learning

Revisit Example With CDL

- Learn, Add, Backjump
- Vastly faster search

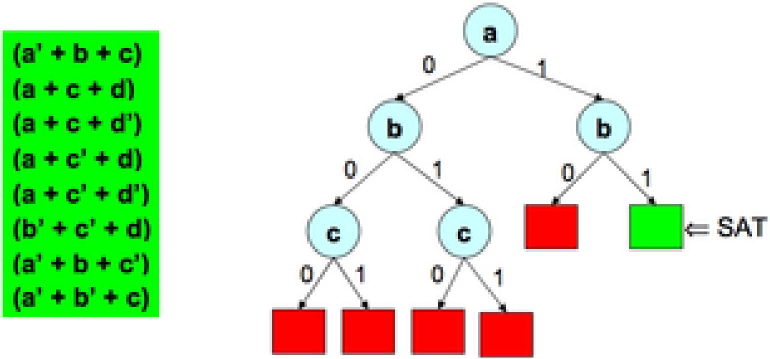


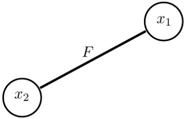
Figure: Boolean Constraint Propagation (Courtesy: Lintao Zhang)

Backtracking Only (01/26)

$$
\begin{array}{ccccc}
\neg x_1 & \vee & x_2 & \vee & x_3 \\
x_1 & \vee & x_3 & \vee & x_4 \\
x_1 & \vee & x_3 & \vee & \neg x_4 \\
x_1 & \vee & \neg x_3 & \vee & x_4 \\
x_1 & \vee & \neg x_3 & \vee & \neg x_4 \\
\neg x_2 & \vee & \neg x_3 & \vee & x_4 \\
\neg x_1 & \vee & x_2 & \vee & \neg x_3 \\
\neg x_1 & \vee & \neg x_2 & \vee & x_3
\end{array}
$$

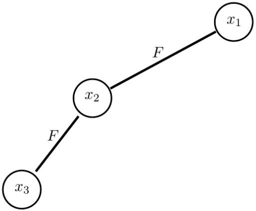
Backtracking Only (02/26)

✓	$\neg x_1$	$\vee$	x_2	$\vee$	x_3
	x_1	$\vee$	x_3	$\vee$	x_4
	x_1	$\vee$	x_3	$\vee$	$\neg x_4$
	x_1	$\vee$	$\neg x_3$	$\vee$	x_4
	x_1	$\vee$	$\neg x_3$	$\vee$	$\neg x_4$
	$\neg x_2$	$\vee$	$\neg x_3$	$\vee$	x_4
✓	$\neg x_1$	$\vee$	x_2	$\vee$	$\neg x_3$
✓	$\neg x_1$	$\vee$	$\neg x_2$	$\vee$	x_3



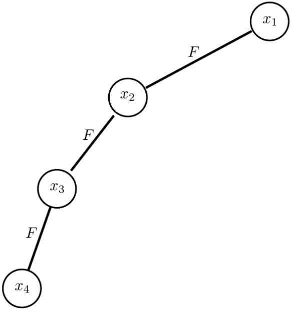
Backtracking Only (03/26)

✓	$\neg x_1$	$\vee$	x_2	$\vee$	x_3
	x_1	$\vee$	x_3	$\vee$	x_4
	x_1	$\vee$	x_3	$\vee$	$\neg x_4$
	x_1	$\vee$	$\neg x_3$	$\vee$	x_4
	x_1	$\vee$	$\neg x_3$	$\vee$	$\neg x_4$
✓	$\neg x_2$	$\vee$	$\neg x_3$	$\vee$	x_4
✓	$\neg x_1$	$\vee$	x_2	$\vee$	$\neg x_3$
✓	$\neg x_1$	$\vee$	$\neg x_2$	$\vee$	x_3



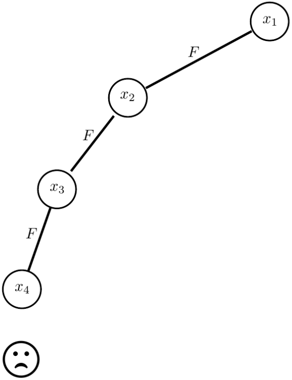
Backtracking Only (04/26)

✓	$\neg x_1$	$\vee$	x_2	$\vee$	x_3
	x_1	$\vee$	x_3	$\vee$	x_4
	x_1	$\vee$	x_3	$\vee$	$\neg x_4$
✓	x_1	$\vee$	$\neg x_3$	$\vee$	x_4
✓	x_1	$\vee$	$\neg x_3$	$\vee$	$\neg x_4$
✓	$\neg x_2$	$\vee$	$\neg x_3$	$\vee$	x_4
✓	$\neg x_1$	$\vee$	x_2	$\vee$	$\neg x_3$
✓	$\neg x_1$	$\vee$	$\neg x_2$	$\vee$	x_3



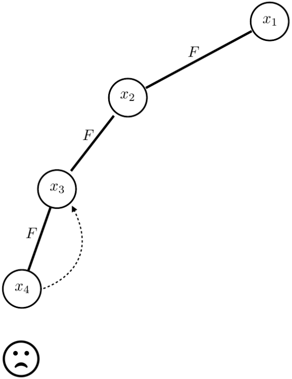
Backtracking Only (05/26)

✓	$\neg x_1$	$\vee$	x_2	$\vee$	x_3
(boxed)	x_1	$\vee$	x_3	$\vee$	x_4
(boxed)	x_1	$\vee$	x_3	$\vee$	$\neg x_4$
✓	x_1	$\vee$	$\neg x_3$	$\vee$	x_4
✓	x_1	$\vee$	$\neg x_3$	$\vee$	$\neg x_4$
✓	$\neg x_2$	$\vee$	$\neg x_3$	$\vee$	x_4
✓	$\neg x_1$	$\vee$	x_2	$\vee$	$\neg x_3$
✓	$\neg x_1$	$\vee$	$\neg x_2$	$\vee$	x_3



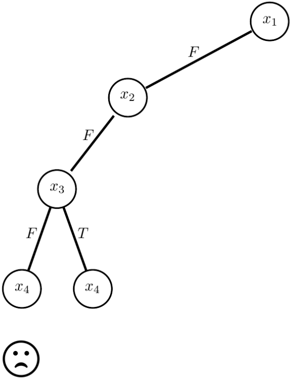
Backtracking Only (06/26)

✓	$\neg x_1$	$\vee$	x_2	$\vee$	x_3
	x_1	$\vee$	x_3	$\vee$	x_4
	x_1	$\vee$	x_3	$\vee$	$\neg x_4$
	x_1	$\vee$	$\neg x_3$	$\vee$	x_4
	x_1	$\vee$	$\neg x_3$	$\vee$	$\neg x_4$
✓	$\neg x_2$	$\vee$	$\neg x_3$	$\vee$	x_4
✓	$\neg x_1$	$\vee$	x_2	$\vee$	$\neg x_3$
✓	$\neg x_1$	$\vee$	$\neg x_2$	$\vee$	x_3



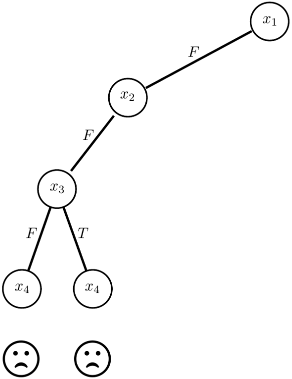
Backtracking Only (07/26)

✓	$\neg x_1$	$\vee$	x_2	$\vee$	x_3
✓	x_1	$\vee$	x_3	$\vee$	x_4
✓	x_1	$\vee$	x_3	$\vee$	$\neg x_4$
	x_1	$\vee$	$\neg x_3$	$\vee$	x_4
	x_1	$\vee$	$\neg x_3$	$\vee$	$\neg x_4$
✓	$\neg x_2$	$\vee$	$\neg x_3$	$\vee$	x_4
✓	$\neg x_1$	$\vee$	x_2	$\vee$	$\neg x_3$
✓	$\neg x_1$	$\vee$	$\neg x_2$	$\vee$	x_3



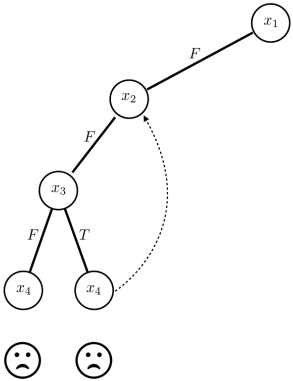
Backtracking Only (08/26)

✓	$\neg x_1$	$\vee$ x_2	$\vee$	x_3	
✓	x_1	$\vee$ x_3	$\vee$	x_4	
✓	x_1	$\vee$ x_3	$\vee$	$\neg x_4$	
	x_1	$\vee$ $\neg x_3$	$\vee$	x_4	
	x_1	$\vee$ $\neg x_3$	$\vee$	$\neg x_4$	
✓	$\neg x_2$	$\vee$ $\neg x_3$	$\vee$	x_4	
✓	$\neg x_1$	$\vee$ x_2	$\vee$	$\neg x_3$	
✓	$\neg x_1$	$\vee$ $\neg x_2$	$\vee$	x_3	



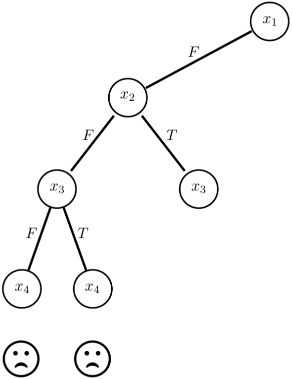
Backtracking Only (09/26)

✓	$\neg x_1$	$\vee$ x_2	$\vee$ x_3		
	x_1	$\vee$ x_3	$\vee$ x_4		
	x_1	$\vee$ x_3	$\vee$ $\neg x_4$		
	x_1	$\vee$ $\neg x_3$	$\vee$ x_4		
	x_1	$\vee$ $\neg x_3$	$\vee$ $\neg x_4$		
	$\neg x_2$	$\vee$ $\neg x_3$	$\vee$ x_4		
✓	$\neg x_1$	$\vee$ x_2	$\vee$ $\neg x_3$		
✓	$\neg x_1$	$\vee$ $\neg x_2$	$\vee$ x_3		



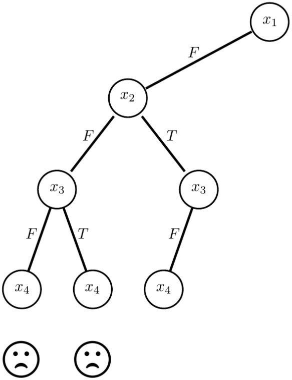
Backtracking Only (10/26)

✓ $\neg x_1 \vee x_2 \vee x_3$
$x_1 \vee x_3 \vee x_4$
$x_1 \vee x_3 \vee \neg x_4$
$x_1 \vee \neg x_3 \vee x_4$
$x_1 \vee \neg x_3 \vee \neg x_4$
$\neg x_2 \vee \neg x_3 \vee x_4$
✓ $\neg x_1 \vee x_2 \vee \neg x_3$
✓ $\neg x_1 \vee \neg x_2 \vee x_3$



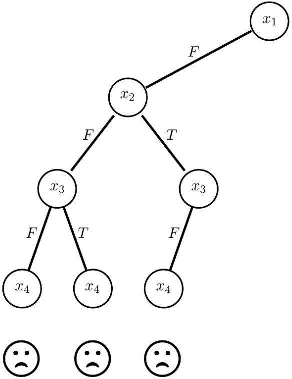
Backtracking Only (11/26)

✓	$\neg x_1$	$\vee$ x_2	$\vee$	x_3	
	x_1	$\vee$ x_3	$\vee$	x_4	
	x_1	$\vee$ x_3	$\vee$	$\neg x_4$	
✓	x_1	$\vee$ $\neg x_3$	$\vee$	x_4	
✓	x_1	$\vee$ $\neg x_3$	$\vee$	$\neg x_4$	
✓	$\neg x_2$	$\vee$ $\neg x_3$	$\vee$	x_4	
✓	$\neg x_1$	$\vee$ x_2	$\vee$	$\neg x_3$	
✓	$\neg x_1$	$\vee$ $\neg x_2$	$\vee$	x_3	



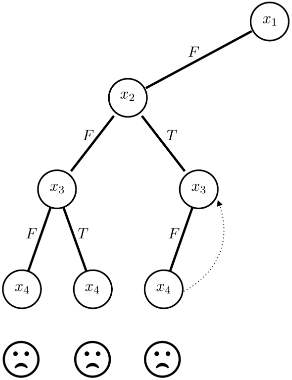
Backtracking Only (12/26)

✓	$\neg x_1 \vee x_2 \vee x_3$
	$x_1 \vee x_3 \vee x_4$
	$x_1 \vee x_3 \vee \neg x_4$
✓	$x_1 \vee \neg x_3 \vee x_4$
✓	$x_1 \vee \neg x_3 \vee \neg x_4$
✓	$\neg x_2 \vee \neg x_3 \vee x_4$
✓	$\neg x_1 \vee x_2 \vee \neg x_3$
✓	$\neg x_1 \vee \neg x_2 \vee x_3$



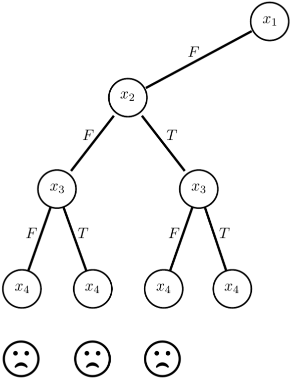
Backtracking Only (13/26)

✓	$\neg x_1$	$\vee$ x_2	$\vee$ x_3		
	x_1	$\vee$ x_3	$\vee$ x_4		
	x_1	$\vee$ x_3	$\vee$ $\neg x_4$		
	x_1	$\vee$ $\neg x_3$	$\vee$ x_4		
	x_1	$\vee$ $\neg x_3$	$\vee$ $\neg x_4$		
	$\neg x_2$	$\vee$ $\neg x_3$	$\vee$ x_4		
✓	$\neg x_1$	$\vee$ x_2	$\vee$ $\neg x_3$		
✓	$\neg x_1$	$\vee$ $\neg x_2$	$\vee$ x_3		



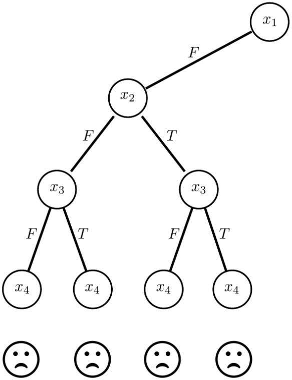
Backtracking Only (14/26)

✓	$\neg x_1$	$\vee$ x_2	$\vee$ x_3		
✓	x_1	$\vee$ x_3	$\vee$ x_4		
✓	x_1	$\vee$ x_3	$\vee$ $\neg x_4$		
	x_1	$\vee$ $\neg x_3$	$\vee$ x_4		
	x_1	$\vee$ $\neg x_3$	$\vee$ $\neg x_4$		
	$\neg x_2$	$\vee$ $\neg x_3$	$\vee$ x_4		
✓	$\neg x_1$	$\vee$ x_2	$\vee$ $\neg x_3$		
✓	$\neg x_1$	$\vee$ $\neg x_2$	$\vee$ x_3		



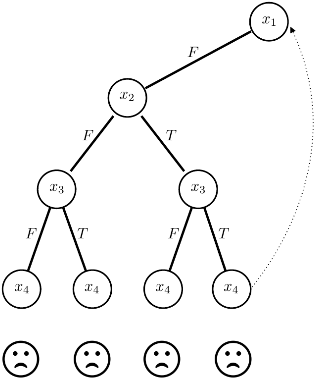
Backtracking Only (15/26)

✓	$\neg x_1$	$\vee$	x_2	$\vee$	x_3
✓	x_1	$\vee$	x_3	$\vee$	x_4
✓	x_1	$\vee$	x_3	$\vee$	$\neg x_4$
	x_1	$\vee$	$\neg x_3$	$\vee$	x_4
	x_1	$\vee$	$\neg x_3$	$\vee$	$\neg x_4$
	$\neg x_2$	$\vee$	$\neg x_3$	$\vee$	x_4
✓	$\neg x_1$	$\vee$	x_2	$\vee$	$\neg x_3$
✓	$\neg x_1$	$\vee$	$\neg x_2$	$\vee$	x_3



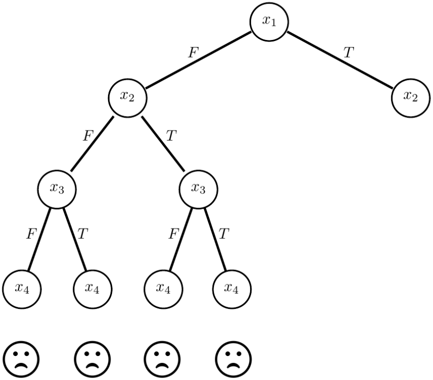
Backtracking Only (16/26)

$\neg x_1$	$\vee$	x_2	$\vee$	x_3
x_1	$\vee$	x_3	$\vee$	x_4
x_1	$\vee$	x_3	$\vee$	$\neg x_4$
x_1	$\vee$	$\neg x_3$	$\vee$	x_4
x_1	$\vee$	$\neg x_3$	$\vee$	$\neg x_4$
$\neg x_2$	$\vee$	$\neg x_3$	$\vee$	x_4
$\neg x_1$	$\vee$	x_2	$\vee$	$\neg x_3$
$\neg x_1$	$\vee$	$\neg x_2$	$\vee$	x_3



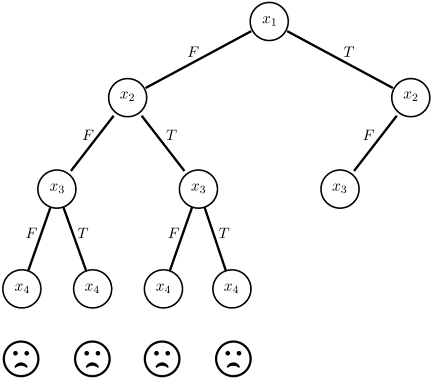
Backtracking Only (17/26)

	$\neg x_1$	$\vee$ x_2	$\vee$	x_3	
✓	x_1	$\vee$ x_3	$\vee$	x_4	
✓	x_1	$\vee$ x_3	$\vee$	$\neg x_4$	
✓	x_1	$\vee$ $\neg x_3$	$\vee$	x_4	
✓	x_1	$\vee$ $\neg x_3$	$\vee$	$\neg x_4$	
	$\neg x_2$	$\vee$ $\neg x_3$	$\vee$	x_4	
	$\neg x_1$	$\vee$ x_2	$\vee$	$\neg x_3$	
	$\neg x_1$	$\vee$ $\neg x_2$	$\vee$	x_3	



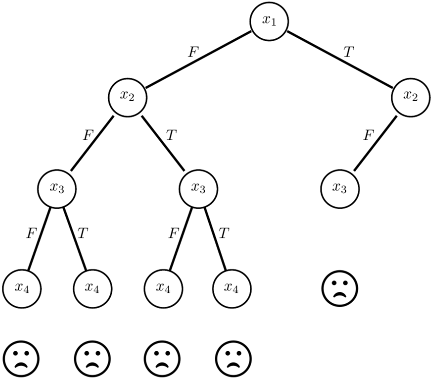
Backtracking Only (18/26)

	$\neg x_1 \vee x_2 \vee x_3$
✓	$x_1 \vee x_3 \vee x_4$
✓	$x_1 \vee x_3 \vee \neg x_4$
✓	$x_1 \vee \neg x_3 \vee x_4$
✓	$x_1 \vee \neg x_3 \vee \neg x_4$
✓	$\neg x_2 \vee \neg x_3 \vee x_4$
	$\neg x_1 \vee x_2 \vee \neg x_3$
✓	$\neg x_1 \vee \neg x_2 \vee x_3$



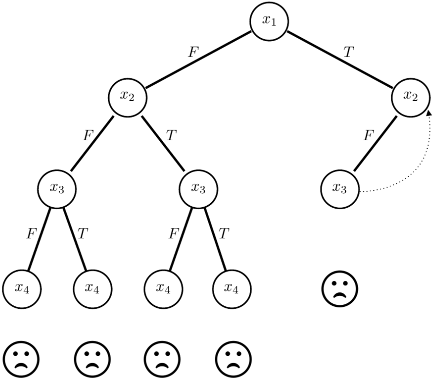
Backtracking Only (19/26)

	Clause
	$\neg x_1 \vee x_2 \vee x_3$
✓	$x_1 \vee x_3 \vee x_4$
✓	$x_1 \vee x_3 \vee \neg x_4$
✓	$x_1 \vee \neg x_3 \vee x_4$
✓	$x_1 \vee \neg x_3 \vee \neg x_4$
✓	$\neg x_2 \vee \neg x_3 \vee x_4$
	$\neg x_1 \vee x_2 \vee \neg x_3$
✓	$\neg x_1 \vee \neg x_2 \vee x_3$



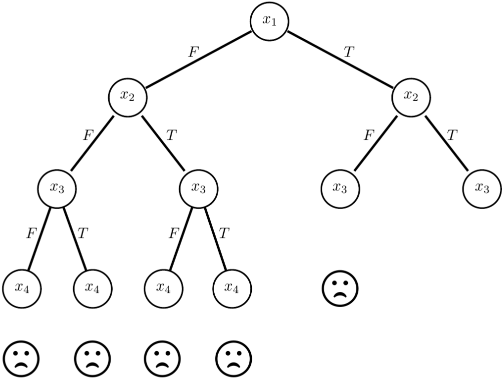
Backtracking Only (20/26)

	$\neg x_1$	$\vee$	x_2	$\vee$	x_3
✓	x_1	$\vee$	x_3	$\vee$	x_4
✓	x_1	$\vee$	x_3	$\vee$	$\neg x_4$
✓	x_1	$\vee$	$\neg x_3$	$\vee$	x_4
✓	x_1	$\vee$	$\neg x_3$	$\vee$	$\neg x_4$
	$\neg x_2$	$\vee$	$\neg x_3$	$\vee$	x_4
	$\neg x_1$	$\vee$	x_2	$\vee$	$\neg x_3$
	$\neg x_1$	$\vee$	$\neg x_2$	$\vee$	x_3



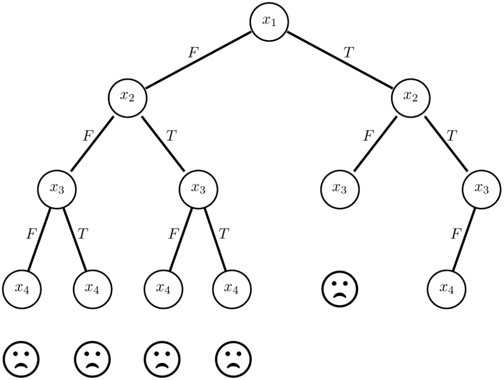
Backtracking Only (21/26)

✓	$\neg x_1$	$\vee$	x_2	$\vee$	x_3
✓	x_1	$\vee$	x_3	$\vee$	x_4
✓	x_1	$\vee$	x_3	$\vee$	$\neg x_4$
✓	x_1	$\vee$	$\neg x_3$	$\vee$	x_4
✓	x_1	$\vee$	$\neg x_3$	$\vee$	$\neg x_4$
	$\neg x_2$	$\vee$	$\neg x_3$	$\vee$	x_4
✓	$\neg x_1$	$\vee$	x_2	$\vee$	$\neg x_3$
	$\neg x_1$	$\vee$	$\neg x_2$	$\vee$	x_3



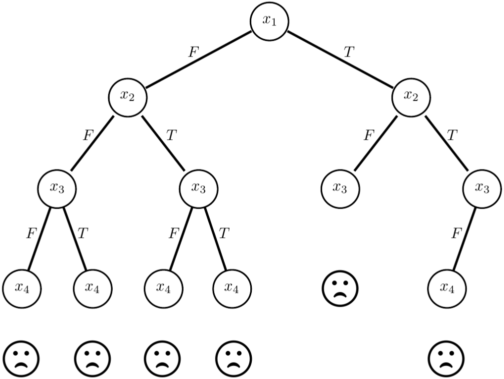
Backtracking Only (22/26)

✓	$\neg x_1$	$\vee$ x_2	$\vee$ x_3		
✓	x_1	$\vee$ x_3	$\vee$ x_4		
✓	x_1	$\vee$ x_3	$\vee$ $\neg x_4$		
✓	x_1	$\vee$ $\neg x_3$	$\vee$ x_4		
✓	x_1	$\vee$ $\neg x_3$	$\vee$ $\neg x_4$		
✓	$\neg x_2$	$\vee$ $\neg x_3$	$\vee$ x_4		
✓	$\neg x_1$	$\vee$ x_2	$\vee$ $\neg x_3$		
	$\neg x_1$	$\vee$ $\neg x_2$	$\vee$ x_3		



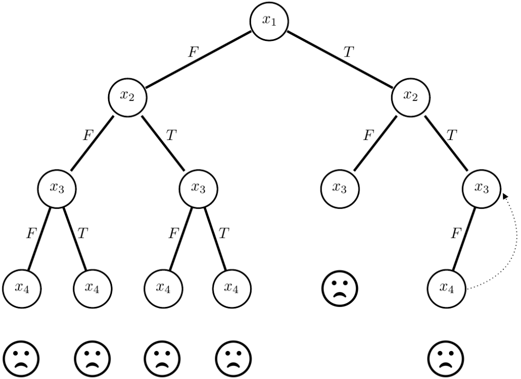
Backtracking Only (23/26)

- ✓ $\neg x_1 \lor x_2 \lor x_3$
- ✓ $x_1 \lor x_3 \lor x_4$
- ✓ $x_1 \lor x_3 \lor \neg x_4$
- ✓ $x_1 \lor \neg x_3 \lor x_4$
- ✓ $x_1 \lor \neg x_3 \lor \neg x_4$
- ✓ $\neg x_2 \lor \neg x_3 \lor x_4$
- ✓ $\neg x_1 \lor x_2 \lor \neg x_3$
- $\boxed{\neg x_1 \lor \neg x_2 \lor x_3}$



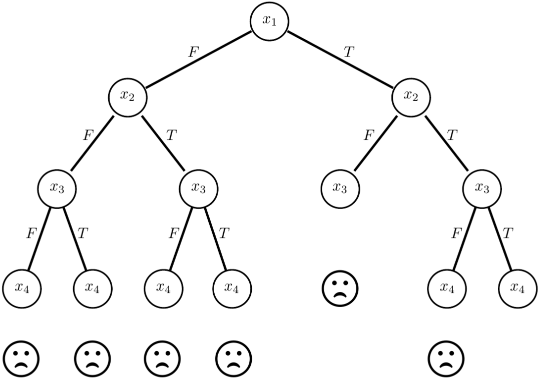
Backtracking Only (24/26)

✓	$\neg x_1$	$\vee$ x_2	$\vee$ x_3		
✓	x_1	$\vee$ x_3	$\vee$ x_4		
✓	x_1	$\vee$ x_3	$\vee$ $\neg x_4$		
✓	x_1	$\vee$ $\neg x_3$	$\vee$ x_4		
✓	x_1	$\vee$ $\neg x_3$	$\vee$ $\neg x_4$		
	$\neg x_2$	$\vee$ $\neg x_3$	$\vee$ x_4		
✓	$\neg x_1$	$\vee$ x_2	$\vee$ $\neg x_3$		
	$\neg x_1$	$\vee$ $\neg x_2$	$\vee$ x_3		



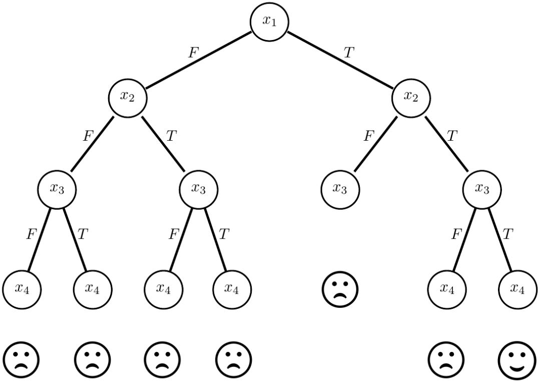
Backtracking Only (25/26)

✓	$\neg x_1$	$\vee$	x_2	$\vee$	x_3
✓	x_1	$\vee$	x_3	$\vee$	x_4
✓	x_1	$\vee$	x_3	$\vee$	$\neg x_4$
✓	x_1	$\vee$	$\neg x_3$	$\vee$	x_4
✓	x_1	$\vee$	$\neg x_3$	$\vee$	$\neg x_4$
	$\neg x_2$	$\vee$	$\neg x_3$	$\vee$	x_4
✓	$\neg x_1$	$\vee$	x_2	$\vee$	$\neg x_3$
✓	$\neg x_1$	$\vee$	$\neg x_2$	$\vee$	x_3



Backtracking Only (26/26)

- ✓ $\neg x_1 \lor x_2 \lor x_3$
- ✓ $x_1 \lor x_3 \lor x_4$
- ✓ $x_1 \lor x_3 \lor \neg x_4$
- ✓ $x_1 \lor \neg x_3 \lor x_4$
- ✓ $x_1 \lor \neg x_3 \lor \neg x_4$
- ✓ $\neg x_2 \lor \neg x_3 \lor x_4$
- ✓ $\neg x_1 \lor x_2 \lor \neg x_3$
- ✓ $\neg x_1 \lor \neg x_2 \lor x_3$

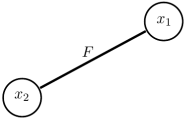


Boolean Constraint Propagation (01/23)

$$
\begin{array}{ccccc}
\neg x_1 & \vee & x_2 & \vee & x_3 \\
x_1 & \vee & x_3 & \vee & x_4 \\
x_1 & \vee & x_3 & \vee & \neg x_4 \\
x_1 & \vee & \neg x_3 & \vee & x_4 \\
x_1 & \vee & \neg x_3 & \vee & \neg x_4 \\
\neg x_2 & \vee & \neg x_3 & \vee & x_4 \\
\neg x_1 & \vee & x_2 & \vee & \neg x_3 \\
\neg x_1 & \vee & \neg x_2 & \vee & x_3
\end{array}
$$

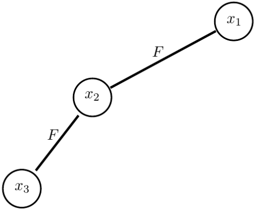
Boolean Constraint Propagation (02/23)

$$
\begin{array}{lccccc}
\checkmark & \neg x_1 & \vee & x_2 & \vee & x_3 \\
 & & & x_3 & \vee & x_4 \\
 & & & x_3 & \vee & \neg x_4 \\
 & & & \neg x_3 & \vee & x_4 \\
 & & & \neg x_3 & \vee & \neg x_4 \\
 & \neg x_2 & \vee & \neg x_3 & \vee & x_4 \\
\checkmark & \neg x_1 & \vee & x_2 & \vee & \neg x_3 \\
\checkmark & \neg x_1 & \vee & \neg x_2 & \vee & x_3
\end{array}
$$



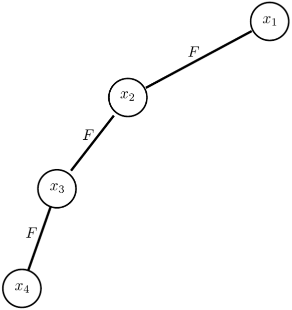
Boolean Constraint Propagation (03/23)

✓	$\neg x_1$	$\vee$ x_2	$\vee$	x_3	
		x_3	$\vee$	x_4	
		x_3	$\vee$	$\neg x_4$	
		$\neg x_3$	$\vee$	x_4	
		$\neg x_3$	$\vee$	$\neg x_4$	
✓	$\neg x_2$	$\vee$ $\neg x_3$	$\vee$	x_4	
✓	$\neg x_1$	$\vee$ x_2	$\vee$	$\neg x_3$	
✓	$\neg x_1$	$\vee$ $\neg x_2$	$\vee$	x_3	



Boolean Constraint Propagation (04/23)

✓	$\neg x_1$	$\vee$	x_2	$\vee$	x_3
					x_4
					$\neg x_4$
✓			$\neg x_3$	$\vee$	x_4
✓			$\neg x_3$	$\vee$	$\neg x_4$
✓	$\neg x_2$	$\vee$	$\neg x_3$	$\vee$	x_4
✓	$\neg x_1$	$\vee$	x_2	$\vee$	$\neg x_3$
✓	$\neg x_1$	$\vee$	$\neg x_2$	$\vee$	x_3



Boolean Constraint Propagation (05/23)

✓ $\neg x_1 \vee x_2 \vee x_3$

x_4

$\neg x_4$

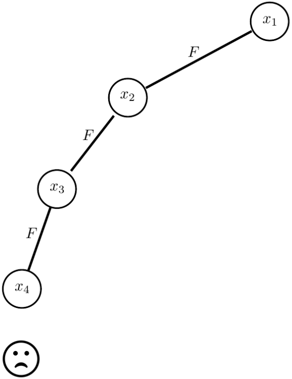
✓ $\neg x_3 \vee x_4$

✓ $\neg x_3 \vee \neg x_4$

✓ $\neg x_2 \vee \neg x_3 \vee x_4$

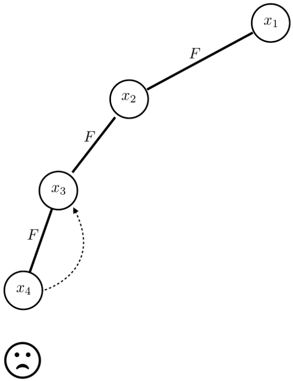
✓ $\neg x_1 \vee x_2 \vee \neg x_3$

✓ $\neg x_1 \vee \neg x_2 \vee x_3$



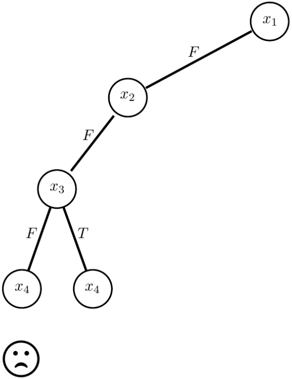
Boolean Constraint Propagation (06/23)

✓	$\neg x_1$	$\vee$	x_2	$\vee$	x_3
			x_3	$\vee$	x_4
			x_3	$\vee$	$\neg x_4$
			$\neg x_3$	$\vee$	x_4
			$\neg x_3$	$\vee$	$\neg x_4$
✓	$\neg x_2$	$\vee$	$\neg x_3$	$\vee$	x_4
✓	$\neg x_1$	$\vee$	x_2	$\vee$	$\neg x_3$
✓	$\neg x_1$	$\vee$	$\neg x_2$	$\vee$	x_3



Boolean Constraint Propagation (07/23)

✓	$\neg x_1$	$\vee$	x_2	$\vee$	x_3
✓			x_3	$\vee$	x_4
✓			x_3	$\vee$	$\neg x_4$
					x_4
					$\neg x_4$
✓	$\neg x_2$	$\vee$	$\neg x_3$	$\vee$	x_4
✓	$\neg x_1$	$\vee$	x_2	$\vee$	$\neg x_3$
✓	$\neg x_1$	$\vee$	$\neg x_2$	$\vee$	x_3



Boolean Constraint Propagation (08/23)

✓ $\neg x_1 \lor x_2 \lor x_3$

✓ $x_3 \lor x_4$

✓ $x_3 \lor \neg x_4$

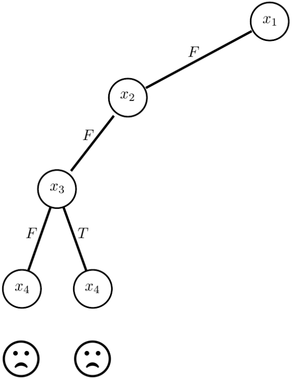
x_4

$\neg x_4$

✓ $\neg x_2 \lor \neg x_3 \lor x_4$

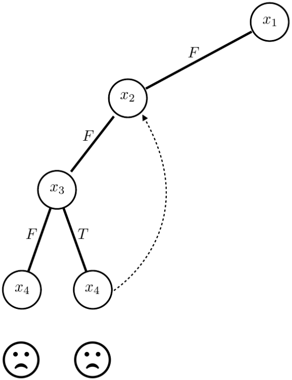
✓ $\neg x_1 \lor x_2 \lor \neg x_3$

✓ $\neg x_1 \lor \neg x_2 \lor x_3$



Boolean Constraint Propagation (09/23)

✓	$\neg x_1$	$\vee$	x_2	$\vee$	x_3
			x_3	$\vee$	x_4
			x_3	$\vee$	$\neg x_4$
			$\neg x_3$	$\vee$	x_4
			$\neg x_3$	$\vee$	$\neg x_4$
	$\neg x_2$	$\vee$	$\neg x_3$	$\vee$	x_4
✓	$\neg x_1$	$\vee$	x_2	$\vee$	$\neg x_3$
✓	$\neg x_1$	$\vee$	$\neg x_2$	$\vee$	x_3



Boolean Constraint Propagation (10/23)

$\checkmark \quad \neg x_1 \vee x_2 \vee x_3$

$x_3 \vee x_4$

$x_3 \vee \neg x_4$

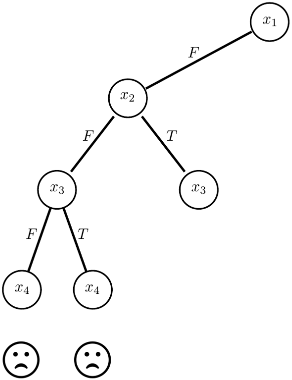
$\neg x_3 \vee x_4$

$\neg x_3 \vee \neg x_4$

$\neg x_3 \vee x_4$

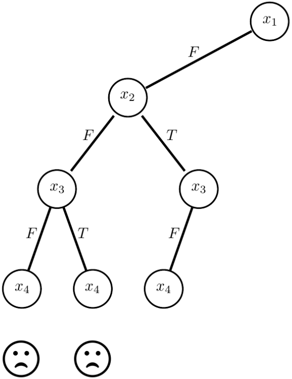
$\checkmark \quad \neg x_1 \vee x_2 \vee \neg x_3$

$\checkmark \quad \neg x_1 \vee \neg x_2 \vee x_3$



Boolean Constraint Propagation (11/23)

✓	$\neg x_1$	$\vee$	x_2	$\vee$	x_3
					x_4
					$\neg x_4$
✓			$\neg x_3$	$\vee$	x_4
✓			$\neg x_3$	$\vee$	$\neg x_4$
✓			$\neg x_3$	$\vee$	x_4
✓	$\neg x_1$	$\vee$	x_2	$\vee$	$\neg x_3$
✓	$\neg x_1$	$\vee$	$\neg x_2$	$\vee$	x_3



Boolean Constraint Propagation (12/23)

✓ $\neg x_1 \lor x_2 \lor x_3$

$\boxed{\begin{array}{c} x_4 \\ \neg x_4 \end{array}}$

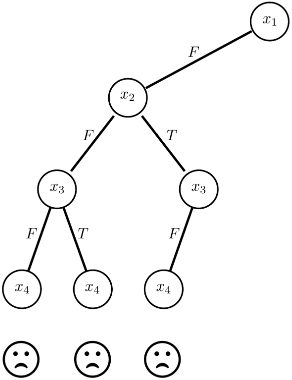
✓ $\neg x_3 \lor x_4$

✓ $\neg x_3 \lor \neg x_4$

✓ $\neg x_3 \lor x_4$

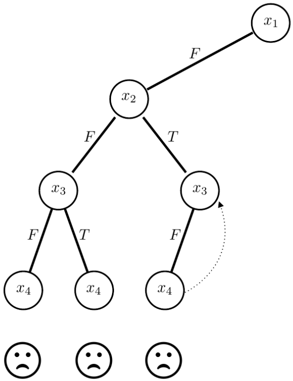
✓ $\neg x_1 \lor x_2 \lor \neg x_3$

✓ $\neg x_1 \lor \neg x_2 \lor x_3$



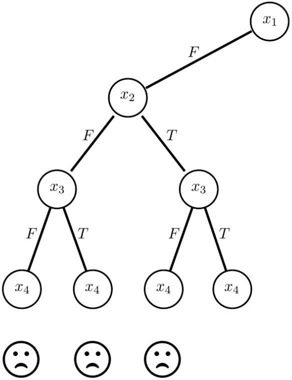
Boolean Constraint Propagation (13/23)

✓	$\neg x_1$	$\vee$	x_2	$\vee$	x_3	
			x_3	$\vee$	x_4	
			x_3	$\vee$	$\neg x_4$	
			$\neg x_3$	$\vee$	x_4	
			$\neg x_3$	$\vee$	$\neg x_4$	
			$\neg x_3$	$\vee$	x_4	
✓	$\neg x_1$	$\vee$	x_2	$\vee$	$\neg x_3$	
✓	$\neg x_1$	$\vee$	$\neg x_2$	$\vee$	x_3	



Boolean Constraint Propagation (14/23)

✓	$\neg x_1$	$\vee$	x_2	$\vee$	x_3
✓			x_3	$\vee$	x_4
✓			x_3	$\vee$	$\neg x_4$
					x_4
					$\neg x_4$
					x_4
✓	$\neg x_1$	$\vee$	x_2	$\vee$	$\neg x_3$
✓	$\neg x_1$	$\vee$	$\neg x_2$	$\vee$	x_3



Boolean Constraint Propagation (15/23)

✓ $\neg x_1 \vee x_2 \vee x_3$

✓ $x_3 \vee x_4$

✓ $x_3 \vee \neg x_4$

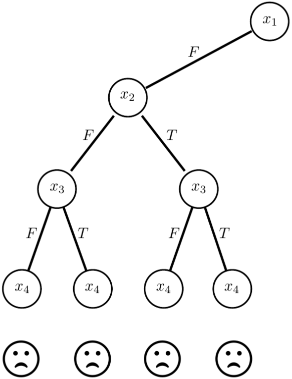
x_4

$\neg x_4$

x_4

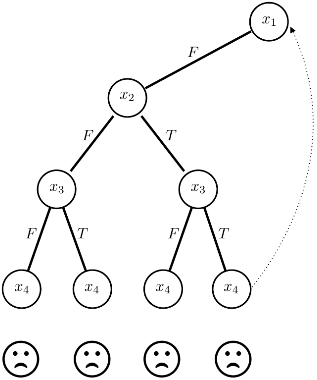
✓ $\neg x_1 \vee x_2 \vee \neg x_3$

✓ $\neg x_1 \vee \neg x_2 \vee x_3$



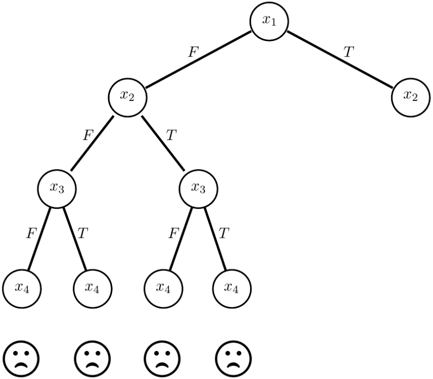
Boolean Constraint Propagation (16/23)

$$
\begin{array}{lllll}
\neg x_1 & \vee & x_2 & \vee & x_3 \\
x_1 & \vee & x_3 & \vee & x_4 \\
x_1 & \vee & x_3 & \vee & \neg x_4 \\
x_1 & \vee & \neg x_3 & \vee & x_4 \\
x_1 & \vee & \neg x_3 & \vee & \neg x_4 \\
\neg x_2 & \vee & \neg x_3 & \vee & x_4 \\
\neg x_1 & \vee & x_2 & \vee & \neg x_3 \\
\neg x_1 & \vee & \neg x_2 & \vee & x_3
\end{array}
$$



Boolean Constraint Propagation (17/23)

		x_2	$\vee$	x_3	
✓	x_1	$\vee$	x_3	$\vee$	x_4
✓	x_1	$\vee$	x_3	$\vee$	$\neg x_4$
✓	x_1	$\vee$	$\neg x_3$	$\vee$	x_4
✓	x_1	$\vee$	$\neg x_3$	$\vee$	$\neg x_4$
	$\neg x_2$	$\vee$	$\neg x_3$	$\vee$	x_4
		x_2	$\vee$	$\neg x_3$	
		$\neg x_2$	$\vee$	x_3	



Boolean Constraint Propagation (18/23)

x_3

✓ $x_1 \lor x_3 \lor x_4$

✓ $x_1 \lor x_3 \lor \neg x_4$

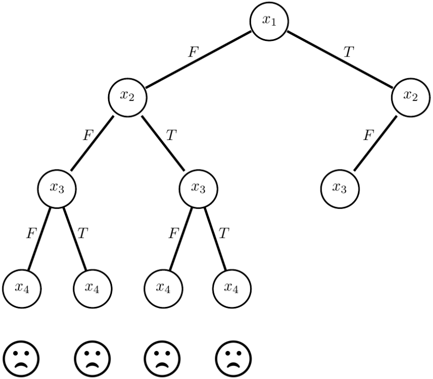
✓ $x_1 \lor \neg x_3 \lor x_4$

✓ $x_1 \lor \neg x_3 \lor \neg x_4$

✓ $\neg x_2 \lor \neg x_3 \lor x_4$

$\neg x_3$

✓ $\neg x_2 \lor x_3$



Boolean Constraint Propagation (19/23)

$\boxed{x_3}$

✓ $x_1 \lor x_3 \lor x_4$

✓ $x_1 \lor x_3 \lor \neg x_4$

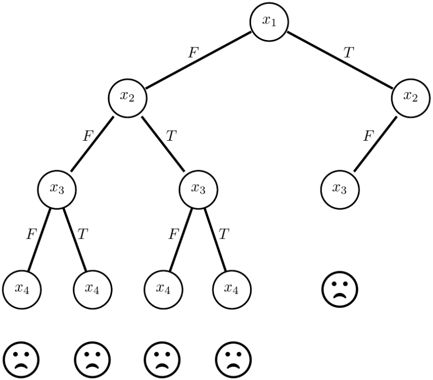
✓ $x_1 \lor \neg x_3 \lor x_4$

✓ $x_1 \lor \neg x_3 \lor \neg x_4$

✓ $\neg x_2 \lor \neg x_3 \lor x_4$

$\boxed{\neg x_3}$

✓ $\neg x_2 \lor x_3$



Boolean Constraint Propagation (20/23)

$x_2 \lor x_3$

✓ $x_1 \lor x_3 \lor x_4$

✓ $x_1 \lor x_3 \lor \neg x_4$

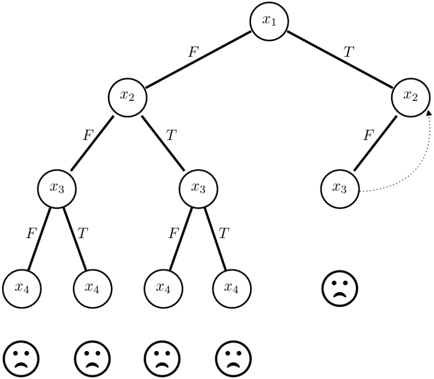
✓ $x_1 \lor \neg x_3 \lor x_4$

✓ $x_1 \lor \neg x_3 \lor \neg x_4$

$\neg x_2 \lor \neg x_3 \lor x_4$

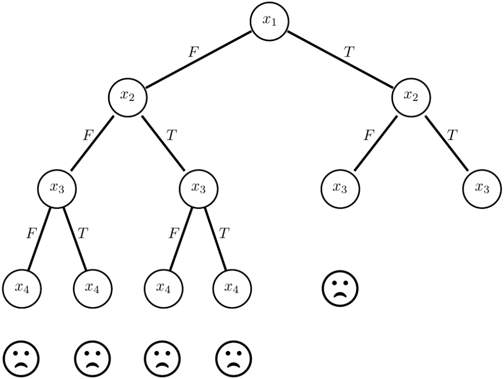
$x_2 \lor \neg x_3$

$\neg x_2 \lor x_3$



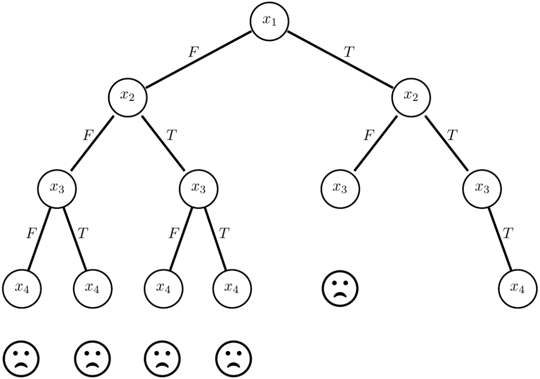
Boolean Constraint Propagation (21/23)

✓	$x_2 \lor x_3$
✓	$x_1 \lor x_3 \lor x_4$
✓	$x_1 \lor x_3 \lor \neg x_4$
✓	$x_1 \lor \neg x_3 \lor x_4$
✓	$x_1 \lor \neg x_3 \lor \neg x_4$
	$\neg x_3 \lor x_4$
✓	$x_2 \lor \neg x_3$
	x_3 ←



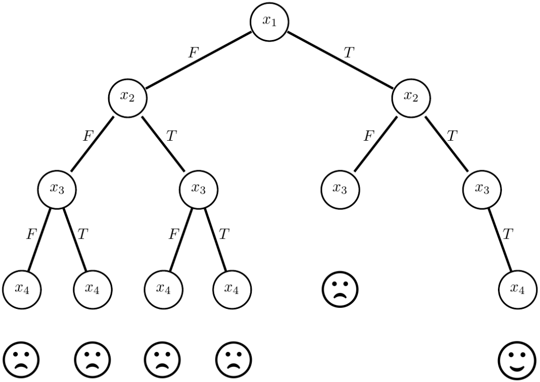
Boolean Constraint Propagation (22/23)

✓			x_2	$\vee$	x_3
✓	x_1	$\vee$	x_3	$\vee$	x_4
✓	x_1	$\vee$	x_3	$\vee$	$\neg x_4$
✓	x_1	$\vee$	$\neg x_3$	$\vee$	x_4
✓	x_1	$\vee$	$\neg x_3$	$\vee$	$\neg x_4$
					x_4
✓			x_2	$\vee$	$\neg x_3$
✓					x_3



Boolean Constraint Propagation (23/23)

✓			x_2	$\vee$	x_3
✓	x_1	$\vee$	x_3	$\vee$	x_4
✓	x_1	$\vee$	x_3	$\vee$	$\neg x_4$
✓	x_1	$\vee$	$\neg x_3$	$\vee$	x_4
✓	x_1	$\vee$	$\neg x_3$	$\vee$	$\neg x_4$
✓					x_4
✓			x_2	$\vee$	$\neg x_3$
✓					x_3

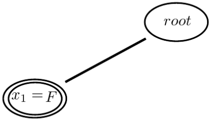


Conflict Driven Learning (01/21)

$$
\begin{array}{lclcl}
\neg x_1 & \vee & x_2 & \vee & x_3 \\
x_1 & \vee & x_3 & \vee & x_4 \\
x_1 & \vee & x_3 & \vee & \neg x_4 \\
x_1 & \vee & \neg x_3 & \vee & x_4 \\
x_1 & \vee & \neg x_3 & \vee & \neg x_4 \\
\neg x_2 & \vee & \neg x_3 & \vee & x_4 \\
\neg x_1 & \vee & x_2 & \vee & \neg x_3 \\
\neg x_1 & \vee & \neg x_2 & \vee & x_3
\end{array}
$$

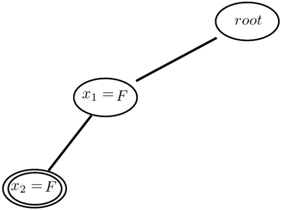
Conflict Driven Learning (02/21)

✓	$\neg x_1 \lor x_2 \lor x_3$
	$x_1 \lor x_3 \lor x_4$
	$x_1 \lor x_3 \lor \neg x_4$
	$x_1 \lor \neg x_3 \lor x_4$
	$x_1 \lor \neg x_3 \lor \neg x_4$
	$\neg x_2 \lor \neg x_3 \lor x_4$
✓	$\neg x_1 \lor x_2 \lor \neg x_3$
✓	$\neg x_1 \lor \neg x_2 \lor x_3$



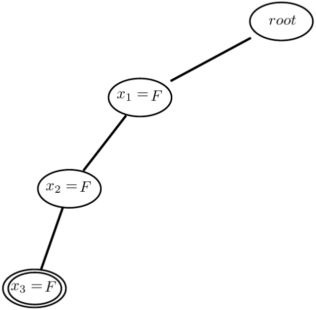
Conflict Driven Learning (03/21)

✓	$\neg x_1$	$\vee$	x_2	$\vee$	x_3
	x_1	$\vee$	x_3	$\vee$	x_4
	x_1	$\vee$	x_3	$\vee$	$\neg x_4$
	x_1	$\vee$	$\neg x_3$	$\vee$	x_4
	x_1	$\vee$	$\neg x_3$	$\vee$	$\neg x_4$
✓	$\neg x_2$	$\vee$	$\neg x_3$	$\vee$	x_4
✓	$\neg x_1$	$\vee$	x_2	$\vee$	$\neg x_3$
✓	$\neg x_1$	$\vee$	$\neg x_2$	$\vee$	x_3



Conflict Driven Learning (04/21)

✓	$\neg x_1$	$\vee$	x_2	$\vee$	x_3
	x_1	$\vee$	x_3	$\vee$	x_4
	x_1	$\vee$	x_3	$\vee$	$\neg x_4$
✓	x_1	$\vee$	$\neg x_3$	$\vee$	x_4
✓	x_1	$\vee$	$\neg x_3$	$\vee$	$\neg x_4$
✓	$\neg x_2$	$\vee$	$\neg x_3$	$\vee$	x_4
✓	$\neg x_1$	$\vee$	x_2	$\vee$	$\neg x_3$
✓	$\neg x_1$	$\vee$	$\neg x_2$	$\vee$	x_3



Conflict Driven Learning (05/21)

✓ $\neg x_1 \lor x_2 \lor x_3$

$x_1 \lor x_3 \lor x_4$

$x_1 \lor x_3 \lor \neg x_4$

✓ $x_1 \lor \neg x_3 \lor x_4$

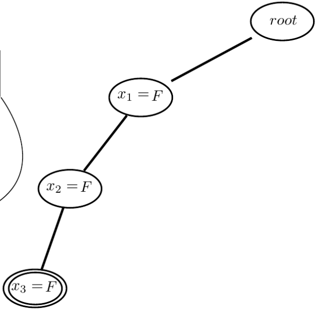
✓ $x_1 \lor \neg x_3 \lor \neg x_4$

✓ $\neg x_2 \lor \neg x_3 \lor x_4$

✓ $\neg x_1 \lor x_2 \lor \neg x_3$

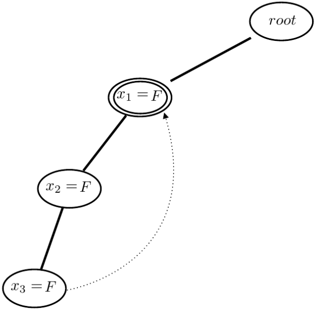
✓ $\neg x_1 \lor \neg x_2 \lor x_3$

$x_1 \lor x_3$



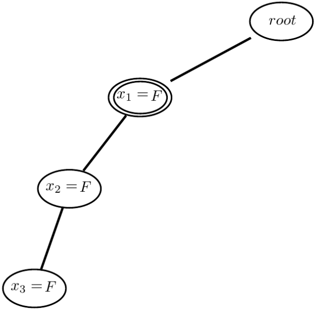
Conflict Driven Learning (06/21)

✓	$\neg x_1$	$\vee$	x_2	$\vee$	x_3
	x_1	$\vee$	x_3	$\vee$	x_4
	x_1	$\vee$	x_3	$\vee$	$\neg x_4$
	x_1	$\vee$	$\neg x_3$	$\vee$	x_4
	x_1	$\vee$	$\neg x_3$	$\vee$	$\neg x_4$
	$\neg x_2$	$\vee$	$\neg x_3$	$\vee$	x_4
✓	$\neg x_1$	$\vee$	x_2	$\vee$	$\neg x_3$
✓	$\neg x_1$	$\vee$	$\neg x_2$	$\vee$	x_3
	x_1	$\vee$	x_3		



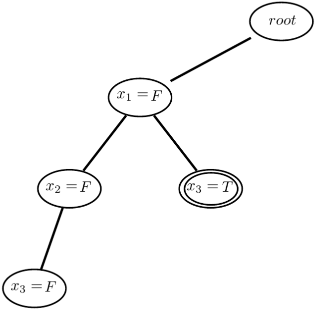
Conflict Driven Learning (07/21)

✓	$\neg x_1 \vee x_2 \vee x_3$
	$x_1 \vee x_3 \vee x_4$
	$x_1 \vee x_3 \vee \neg x_4$
	$x_1 \vee \neg x_3 \vee x_4$
	$x_1 \vee \neg x_3 \vee \neg x_4$
	$\neg x_2 \vee \neg x_3 \vee x_4$
✓	$\neg x_1 \vee x_2 \vee \neg x_3$
✓	$\neg x_1 \vee \neg x_2 \vee x_3$
	$x_1 \vee x_3$ ←



Conflict Driven Learning (08/21)

✓	$\neg x_1$	$\vee$	x_2	$\vee$	x_3
✓	x_1	$\vee$	x_3	$\vee$	x_4
✓	x_1	$\vee$	x_3	$\vee$	$\neg x_4$
	x_1	$\vee$	$\neg x_3$	$\vee$	x_4
	x_1	$\vee$	$\neg x_3$	$\vee$	$\neg x_4$
	$\neg x_2$	$\vee$	$\neg x_3$	$\vee$	x_4
✓	$\neg x_1$	$\vee$	x_2	$\vee$	$\neg x_3$
✓	$\neg x_1$	$\vee$	$\neg x_2$	$\vee$	x_3
✓	x_1	$\vee$	x_3		



Conflict Driven Learning (09/21)

✓ $\neg x_1 \vee x_2 \vee x_3$

✓ $x_1 \vee x_3 \vee x_4$

✓ $x_1 \vee x_3 \vee \neg x_4$

$x_1 \vee \neg x_3 \vee x_4$

$x_1 \vee \neg x_3 \vee \neg x_4$

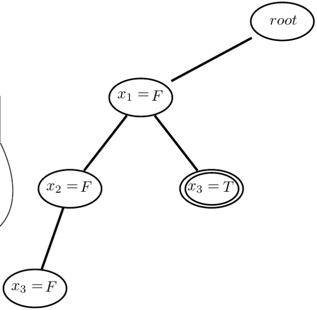
$\neg x_2 \vee \neg x_3 \vee x_4$

✓ $\neg x_1 \vee x_2 \vee \neg x_3$

✓ $\neg x_1 \vee \neg x_2 \vee x_3$

✓ $x_1 \vee x_3$

$x_1 \vee \neg x_3$



Conflict Driven Learning (10/21)

✓ $\neg x_1 \vee x_2 \vee x_3$

$x_1 \vee x_3 \vee x_4$

$x_1 \vee x_3 \vee \neg x_4$

$x_1 \vee \neg x_3 \vee x_4$

$x_1 \vee \neg x_3 \vee \neg x_4$

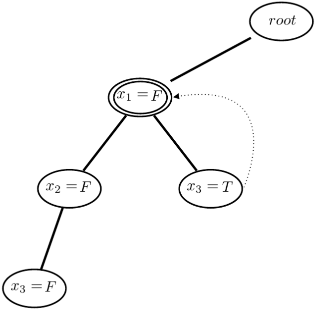
$\neg x_2 \vee \neg x_3 \vee x_4$

✓ $\neg x_1 \vee x_2 \vee \neg x_3$

✓ $\neg x_1 \vee \neg x_2 \vee x_3$

$x_1 \vee x_3$

$x_1 \vee \neg x_3$



Conflict Driven Learning (11/21)

✓ $\neg x_1 \vee x_2 \vee x_3$

$x_1 \vee x_3 \vee x_4$

$x_1 \vee x_3 \vee \neg x_4$

$x_1 \vee \neg x_3 \vee x_4$

$x_1 \vee \neg x_3 \vee \neg x_4$

$\neg x_2 \vee \neg x_3 \vee x_4$

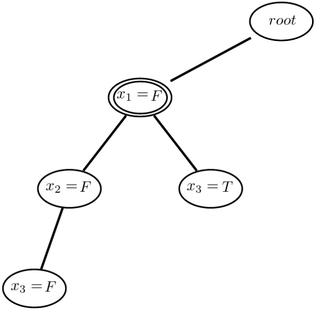
✓ $\neg x_1 \vee x_2 \vee \neg x_3$

✓ $\neg x_1 \vee \neg x_2 \vee x_3$

$x_1 \vee x_3$

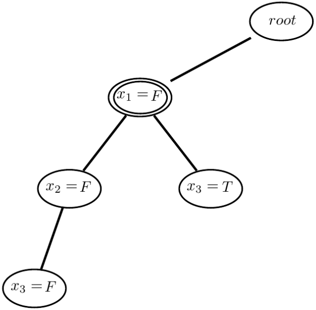
$x_1 \vee \neg x_3$

x_1



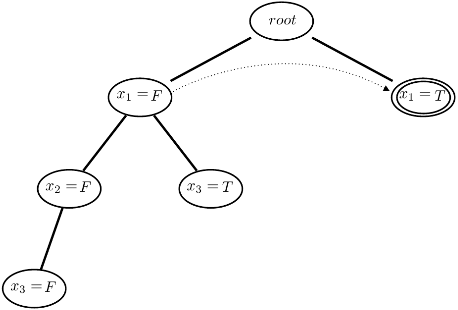
Conflict Driven Learning (12/21)

✓	$\neg x_1 \lor x_2 \lor x_3$	
	$x_1 \lor x_3 \lor x_4$	
	$x_1 \lor x_3 \lor \neg x_4$	
	$x_1 \lor \neg x_3 \lor x_4$	
	$x_1 \lor \neg x_3 \lor \neg x_4$	
	$\neg x_2 \lor \neg x_3 \lor x_4$	
✓	$\neg x_1 \lor x_2 \lor \neg x_3$	
✓	$\neg x_1 \lor \neg x_2 \lor x_3$	
	$x_1 \lor x_3$	
	$x_1 \lor \neg x_3$	
	x_1	←



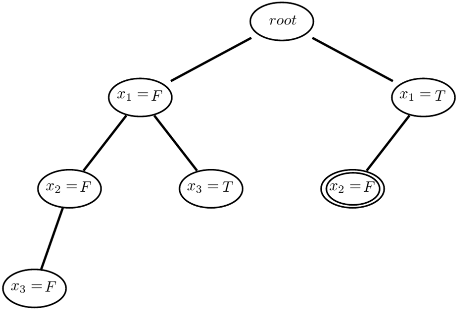
Conflict Driven Learning (13/21)

	$\neg x_1$	$\vee$	x_2	$\vee$	x_3
✓	x_1	$\vee$	x_3	$\vee$	x_4
✓	x_1	$\vee$	x_3	$\vee$	$\neg x_4$
✓	x_1	$\vee$	$\neg x_3$	$\vee$	x_4
✓	x_1	$\vee$	$\neg x_3$	$\vee$	$\neg x_4$
	$\neg x_2$	$\vee$	$\neg x_3$	$\vee$	x_4
	$\neg x_1$	$\vee$	x_2	$\vee$	$\neg x_3$
	$\neg x_1$	$\vee$	$\neg x_2$	$\vee$	x_3
✓	x_1	$\vee$	x_3		
✓	x_1	$\vee$	$\neg x_3$		
✓	x_1				



Conflict Driven Learning (14/21)

$\neg x_1 \lor x_2 \lor x_3$
✓ $x_1 \lor x_3 \lor x_4$
✓ $x_1 \lor x_3 \lor \neg x_4$
✓ $x_1 \lor \neg x_3 \lor x_4$
✓ $x_1 \lor \neg x_3 \lor \neg x_4$
✓ $\neg x_2 \lor \neg x_3 \lor x_4$
$\neg x_1 \lor x_2 \lor \neg x_3$
✓ $\neg x_1 \lor \neg x_2 \lor x_3$
✓ $x_1 \lor x_3$
✓ $x_1 \lor \neg x_3$
✓ x_1



Conflict Driven Learning (15/21)

$\neg x_1 \lor x_2 \lor x_3$

✓ $x_1 \lor x_3 \lor x_4$

✓ $x_1 \lor x_3 \lor \neg x_4$

✓ $x_1 \lor \neg x_3 \lor x_4$

✓ $x_1 \lor \neg x_3 \lor \neg x_4$

✓ $\neg x_2 \lor \neg x_3 \lor x_4$

$\neg x_1 \lor x_2 \lor \neg x_3$

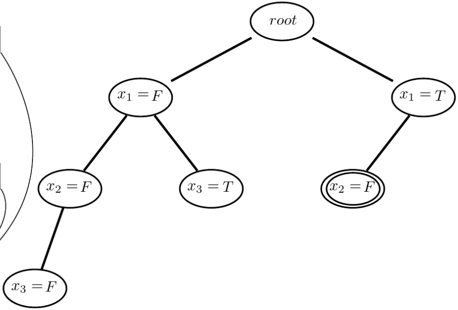
✓ $\neg x_1 \lor \neg x_2 \lor x_3$

✓ $x_1 \lor x_3$

✓ $x_1 \lor \neg x_3$

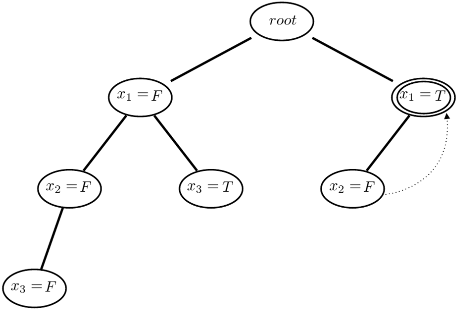
✓ x_1

$\neg x_1 \lor x_2$



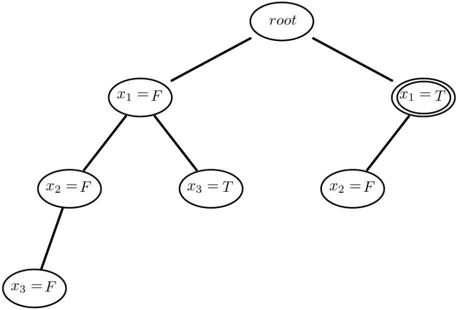
Conflict Driven Learning (16/21)

	$\neg x_1 \lor x_2 \lor x_3$
✓	$x_1 \lor x_3 \lor x_4$
✓	$x_1 \lor x_3 \lor \neg x_4$
✓	$x_1 \lor \neg x_3 \lor x_4$
✓	$x_1 \lor \neg x_3 \lor \neg x_4$
	$\neg x_2 \lor \neg x_3 \lor x_4$
	$\neg x_1 \lor x_2 \lor \neg x_3$
	$\neg x_1 \lor \neg x_2 \lor x_3$
✓	$x_1 \lor x_3$
✓	$x_1 \lor \neg x_3$
✓	x_1
	$\neg x_1 \lor x_2$



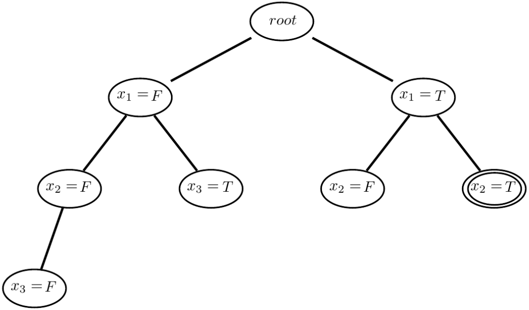
Conflict Driven Learning (17/21)

	$\neg x_1$	$\vee$	x_2	$\vee$	x_3	
✓	x_1	$\vee$	x_3	$\vee$	x_4	
✓	x_1	$\vee$	x_3	$\vee$	$\neg x_4$	
✓	x_1	$\vee$	$\neg x_3$	$\vee$	x_4	
✓	x_1	$\vee$	$\neg x_3$	$\vee$	$\neg x_4$	
	$\neg x_2$	$\vee$	$\neg x_3$	$\vee$	x_4	
	$\neg x_1$	$\vee$	x_2	$\vee$	$\neg x_3$	
	$\neg x_1$	$\vee$	$\neg x_2$	$\vee$	x_3	
✓	x_1	$\vee$	x_3			
✓	x_1	$\vee$	$\neg x_3$			
✓	x_1					
	$\neg x_1$	$\vee$	x_2	←		



Conflict Driven Learning (18/21)

✓	$\neg x_1 \vee x_2 \vee x_3$
✓	$x_1 \vee x_3 \vee x_4$
✓	$x_1 \vee x_3 \vee \neg x_4$
✓	$x_1 \vee \neg x_3 \vee x_4$
✓	$x_1 \vee \neg x_3 \vee \neg x_4$
	$\neg x_2 \vee \neg x_3 \vee x_4$
✓	$\neg x_1 \vee x_2 \vee \neg x_3$
	$\neg x_1 \vee \neg x_2 \vee x_3$
✓	$x_1 \vee x_3$
✓	$x_1 \vee \neg x_3$
✓	x_1
✓	$\neg x_1 \vee x_2$



Conflict Driven Learning (19/21)

✓ $\neg x_1 \lor x_2 \lor x_3$

✓ $x_1 \lor x_3 \lor x_4$

✓ $x_1 \lor x_3 \lor \neg x_4$

✓ $x_1 \lor \neg x_3 \lor x_4$

✓ $x_1 \lor \neg x_3 \lor \neg x_4$

$\neg x_2 \lor \neg x_3 \lor x_4$

✓ $\neg x_1 \lor x_2 \lor \neg x_3$

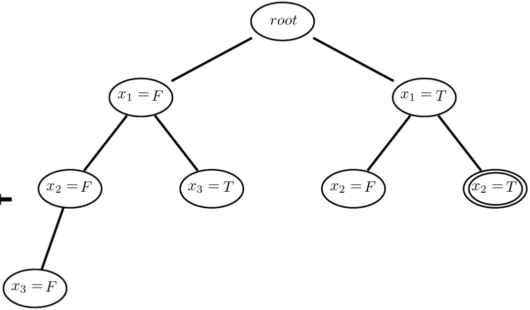
$\neg x_1 \lor \neg x_2 \lor x_3$ ←

✓ $x_1 \lor x_3$

✓ $x_1 \lor \neg x_3$

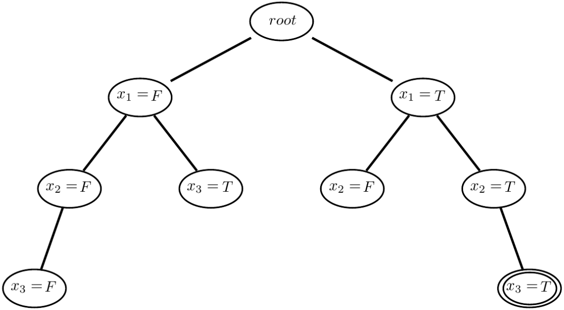
✓ x_1

✓ $\neg x_1 \lor x_2$



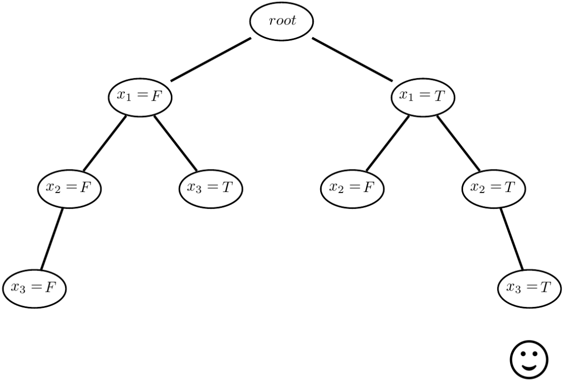
Conflict Driven Learning (20/21)

✓ $\neg x_1 \vee x_2 \vee x_3$
✓ $x_1 \vee x_3 \vee x_4$
✓ $x_1 \vee x_3 \vee \neg x_4$
✓ $x_1 \vee \neg x_3 \vee x_4$
✓ $x_1 \vee \neg x_3 \vee \neg x_4$
$\neg x_2 \vee \neg x_3 \vee x_4$
✓ $\neg x_1 \vee x_2 \vee \neg x_3$
✓ $\neg x_1 \vee \neg x_2 \vee x_3$
✓ $x_1 \vee x_3$
✓ $x_1 \vee \neg x_3$
✓ x_1
✓ $\neg x_1 \vee x_2$



Conflict Driven Learning (21/21)

✓ $\neg x_1 \vee x_2 \vee x_3$
✓ $x_1 \vee x_3 \vee x_4$
✓ $x_1 \vee x_3 \vee \neg x_4$
✓ $x_1 \vee \neg x_3 \vee x_4$
✓ $x_1 \vee \neg x_3 \vee \neg x_4$
✓ $\neg x_2 \vee \neg x_3 \vee x_4$
✓ $\neg x_1 \vee x_2 \vee \neg x_3$
✓ $\neg x_1 \vee \neg x_2 \vee x_3$
✓ $x_1 \vee x_3$
✓ $x_1 \vee \neg x_3$
✓ x_1
✓ $\neg x_1 \vee x_2$



More Details about SAT Solvers

Lectures By Lintao Zhang (ZChaff)

- 1
- 2

Next Time: SMT = SAT + Theories

1. Propositional Logic
2. Combining Theories
   - *Equality + Uninterpreted Functions*
   - *Difference-Bounded Arithmetic*
3. Combining SAT + Theories