Floyd-Hoare Logic & Verification Conditions

Ranjit Jhala, UC San Diego

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A Small Imperative Language

```
data Var

data Exp

data Pred
```

A Small Imperative Language

```
data Com = Asgn  Var Expr
         | Seq   Com Com
         | If    Exp Com Com
         | While Pred Exp Com
         | Skip
```

Verification Condition Generation

Use the `State` monad to log individual loop invariant requirements

```
type VC = State [Pred]  -- validity queries for SMT solver
```

Top Level Verification Function

The top level verifier, takes:

- **Input** : precondition `p`, command `c` andpostcondition `q`
- **Output** : `True` iff $\{\texttt{p}\}$ `c` $\{\texttt{q}\}$ is a valid Hoare-Triple

```
verify          :: Pred -> Com -> Pred -> Bool

verify p c q     = all smtValid queries
  where
    (q', conds) = runState (vcgen q c) []
    queries     = p `implies` q' : conds
```

Verification Condition Generator

```
vcgen :: Pred -> Com -> VC Pred

vcgen (Skip) q
  = return q
vcgen (Asgn x e) q
  = return $ q `subst` (x, e)
vcgen (Seq s1 s2) q
  = vcgen s1 =<< vcgen s2 q
vcgen (If b c1 c2) q
  = do q1    <- vcgen c1 q
       q2    <- vcgen c2 q
       return $ (b `implies` q1) `And` (Not b `implies` q2)
vcgen (While i b c) q
  = do q'            <- vcgen c i
       sideCondition $ (i `And` b)     `implies` q'
       sideCondition $ (i `And` Not b) `implies` q
       return $ i
```

vcgen Helper Logs All Side Conditions

```
sideCond :: Pred -> VC ()
sideCond p = modify $ \conds -> p : conds
```

Next: Some Examples

Now, lets *use* the above verifier to check some programs

Example 1

Consider the program c defined:

```
while (x > 0) {
  x = x - 1;
  y = y - 2;
}
```

Lets prove that

```
{x==8 && y==16} c {y == 0}
```

Example 1

Add the pre- and post-condition with `assume` and `assert`

```
assume(x == 8 && y == 16);
while (x > 0) {
  x = x - 1;
  y = y - 2;
}
assert(y == 0);
```

What do we need next?

Example 1: Adding A Loop Invariant

Lets use a *placeholder* `I` for the invariant

```
assume(x == 8 && y == 16);
while (x > 0) {
  invariant(I);
  x = x - 1;
  y = y - 2;
}
assert(y == 0);
```

Question: What should `I` be?

1. **Weak** enough to hold *initially*
2. **Inductive** to prove *preservation*
3. **Strong** enough to prove *goal*

Example 1: Adding A Loop Invariant

Lets try the candidate invariant `y == 2 * x`

```
assume(x == 8 && y == 16);
while (x > 0) {
  invariant(y == 2 * x);
  x = x - 1;
  y = y - 2;
}
assert(y == 0);
```

1. Holds initially?
   - SMT-Valid `(x == 8 && y == 16) => (y == 2 * x)` ?
   - **[Yes]**

Example 1: Adding A Loop Invariant

Lets try the candidate invariant `y == 2 * x`

```
assume(x == 8 && y == 16);
while (x > 0) {
  invariant(y == 2 * x);
  x = x - 1;
  y = y - 2;
}
assert(y == 0);
```

2. Preserved ?
   - SMT-Valid `(y = 2 * x && x > 0) => (y-2 == 2 * (x - 1))` ?
   - **[Yes]**

Example 1: Adding A Loop Invariant

Lets try the candidate invariant `y == 2 * x`

```
assume(x == 8 && y == 16);
while (x > 0) {
  invariant(y == 2 * x);
  x = x - 1;
  y = y - 2;
}
assert(y == 0);
```

3. Strong Enough To Prove Goal?
   - SMT-Valid `(y = 2 * x && !x > 0) => (y == 0)` ?
   - **[No]**

Uh oh. Close, but no cigar...

Example 1: Adding A Loop Invariant (Take 2)

Lets try `(y == 2 * x) && (x >=0)`

```
assume(x == 8 && y == 16);
while (x > 0) {
  invariant(y == 2 * x && x >= 0);
  x = x - 1;
  y = y - 2;
}
assert(y == 0);
```

SMT Valid Check

1. **Initial** `(x == 8 && y == 16) => (y == 2 * x)`
   - **Yes**
2. **Preserve** `(y = 2 * x && x > 0) => (y-2 == 2 * (x - 1))`
   - **Yes**
3. **Goal** `(y = 2 * x && x >=0 && !x > 0) => (y == 0)`
   - **Yes**

Example 2

```
assume(n > 0);
var k = 0;
var r = 0;
var s = 1;
while (k != n) {
  invariant(I);
  r = r + s;
  s = s + 2;
  k = k + 1;
}
assert(r == n * n);
```

Whoa! What's a reasonable invariant `I`?

Example 2

Lets try the obvious thing . . . `r == k * k`

```
assume(n > 0);
var k = 0;
var r = 0;
var s = 1;
while (k != n) {
  invariant(r == k * k);
  r = r + s;
  s = s + 2;
  k = k + 1;
}
assert(r == n * n);
```

- **Initial** `(k == 0 && r == 0) => (r == k * k)` **YES**
- **Goal** `(r == k * k && k == n) => (r == n * n)` **YES**
- **Preserve** `(r== k*k && k != n) => (r + s == (k+1)*(k+1))` **NO!**

Example 2

Finding an invariant that is **preserved** can be tricky...

... typically need to **strengthen** to get preservation

... that is, to **add extra** conjuncts

Example 2: Take 2

Strengthen I with facts about s

```
assume(n > 0);
var k = 0;
var r = 0;
var s = 1;
while (k != n) {
  invariant(r == k*k && s == 2*k + 1);
  r = r + s;
  s = s + 2;
  k = k + 1;
}
assert(r == n * n);
```

1. **Initial**
   - `(k == 0 && r == 0 && s==1) => (r == k*k && s == 2*k + 1)`
   - **YES**

Example 2: Take 2

Strengthen I with facts about s

```
assume(n > 0);
var k = 0;
var r = 0;
var s = 1;
while (k != n) {
  invariant(r == k*k && s == 2*k + 1);
  r = r + s;
  s = s + 2;
  k = k + 1;
}
assert(r == n * n);
```

2. **Goal**

- `(r == k*k && s == 2*k + 1 && k == n) => (r == n*n)`
- **YES**

Example 2: Take 2

Strengthen I with facts about s

```
assume(n > 0);
var k = 0;
var r = 0;
var s = 1;
while (k != n) {
  invariant(r == k*k && s == 2*k + 1);
  r = r + s;
  s = s + 2;
  k = k + 1;
}
assert(r == n * n);
```

3. **Preserve**

```
(r == k * k && s == 2 * k + 1 && k != n)
  =>
(r + s == (k+1) * (k+1) && s+2 == 2 * (k+1) + 1)
```

Adding Features To IMP

- **Functions**
- Pointers

IMP + Functions

```
data Fun = F String [Var] Com

data Com = ...
         | Call   Var Fun [Expr]
         | Return Expr

data Pgm = [Fun]
```

IMP + Functions

A *function* is a big sequence of `Com` which **does not modify** formals

```
function f(x1,...,xn){
  requires(pre);
  ensures(post);
  body;
  return e;
}
```

Precondition

- Predicate over the **formal** parameters `x1,...,xn`
- That records **assumption** about inputs

Postcondition

- Predicate over the **formals** and **return value** `$result`
- That records **assertion** about outputs

Modular Verification With Contracts

- Together, *pre-* and *post-* conditions called **contracts**
- We can generate VC (hence, verify) **one-function-at-a-time**
- Using just *contracts* for all called functions

Questions

1. How to verify *each* function with *callee* contracts?
2. How to verify `Call` commands?

Verifying A Single Function

To verify a single function

```
function f(x1,...,xn){
  requires(pre);
  ensures(post);
  body;
  return e;
}
```

we need to just verify the Hoare-triple

{pre} `body ; $result := r` {post}

Exercise How will you handle `return` sprinkled within `body` ?

Verifying A Single Call Command

To establish a Hoare-triple for a single **call** command

```
{P}
   y := f(e)
{Q}
```

1. We must **guarantee** that `pre` (of `f`) holds *before* the call
2. We can **assume** that `post (off‘)` holds *after* the call

Hence, the above triple reduces to verifying that

```
{P}
   assert (pre[e1/x1,...,en/xn]) ;
   assume (post[e1/x1,...,en/xn, tmp/$result];
   y := tmp;
{Q}
```

where `tmp` is a fresh temporary variable.

Caller-Callee Contract Duality

Note that at the **callsite** for a function, we

- **assert** the pre-condition
- **assume** the post-condition

while when checking the **callee** we

- **assume** the pre-condition
- **assert** the post-condition

This is key for **modular verification**

- Breaks verification up into pieces matching function abstraction

Example

Consider a function

```
function binarySearch(a, v){
  requires(sorted(a));
  ensures($result == -1
          || 0 <= $result < a.length && a[$result] == v
          );
  ...
}
```

where we want to verify

```
assume(sorted(arr));
y = binarySearch(arr, 12);
if (y != -1){
  assert (arr[y] == 12)
  ...
}
```

Example: Precondition VC

Consider a function

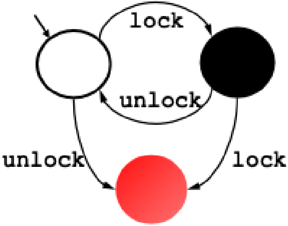
```
function binarySearch(a, v){
  requires(sorted(a));
  ensures($result == -1
          || 0 <= $result < a.length && a[$result] == v
          );
  ...
}
```

Replace call with `assert` and `assume`

```
//pre[arr/a, 12/v]
assert(sorted(arr));

//post[arr/a, 12/v, y/£result]
assume(y==-1
      || 0<=y<a.length && a[y] == 12);
```

Example: A Locking Protocol



*"An attempt to re-acquire an acquired lock or release a released lock will cause a **deadlock**."*

Figure: Calls to `lock` and `unlock` Must Alternate

Example: A Locking Protocol

The `lock` and `unlock` functions

```
function lock(l){
  assert(l == 0); //UNLOCKED
  return 1;       //LOCKED
}

function unlock(l){
  assert(l == 1); //UNLOCKED
  return 0;        //LOCKED
}
```

State of lock encoded in value

What are the **contracts** ? Pretty easy. . .

Example: A Locking Protocol

The `lock` and `unlock` functions with contracts

```
function lock(l){
  requires(l == 0);
  ensures($result == 1);

  assert(l == 0); //UNLOCKED
  return 1;       //LOCKED
}

function unlock(l){
  requires(l == 1);
  ensures($result == 0);

  assert(l == 1); //UNLOCKED
  return 0;        //LOCKED
}
```

Example: Lock Verification

To verify this program

```
assume(l == 0);
if (n % 2 == 0) {
  l = lock(l);
}
...
if (n % 2 == 0) {
  l = unlock(l);
}
```

we just verify

```
assume(l == 0);
if (n % 2 == 0) {
  assert(l == 0);
  assume(tmpa == 1);
  l = tmpa;
}
```

Adding Features To IMP

- Functions
- **Pointers**

IMP + Pointers

Let us add references to IMP

```
data Com = Deref      Var Var        --  x := *y
         | DerefAsgn Var Expr        -- *x := e
```

We find that our **assignment** rule **does not work** with **aliasing**

Assignments and Aliasing

As `*x` and `*y` are aliased, the following is valid

```
{x == y}    *x = 5    {*x + *y == 10}
```

Assignments and Aliasing

In general, for what `P` is the following valid?

`{P}     *x = 5    {*x + *y == 10}`

Intuitively, `P` is something like

`*y == 5 || x = y`

- In the first case, the two sum upto 10.
- In the second case, the aliasing kicks in.

Assignments and Aliasing

In general, for what `P` is the following valid?

`{P}     *x = 5    {*x + *y == 10}`

But the Hoare-rule gives us

`(*x + *y == 10)[5 / *x]`

`== (5 + *y == 10)`

`== (*y == 5)`

Assignments and Aliasing

Uh oh! We lost one case! What happened?!

The substitution `[e/x]` only works when

- `x` is the **only** representation for the value in the predicate

Here, there were **two possible** representations

- `*x`
- `*y`

and we say *possible* because it depends on the aliasing.

- This is why **aliasing is tricky**

Verification With References

Key idea

Beef up our logic to handle memory as a *monolithic entity*

1. Extend **logic theory** (and SMT solver)
2. Extend **Hoare-Rule**

Note: Classical solution due to McCarthy

- It has its issues but thats another story. . .

A Logic For Modelling References

1. Memory **Variables** `M :: Mem`
2. **Select** Operator for reading memory `sel`
3. **Update** Operator for writing memory `upd`
4. **Axioms** for reasoning about `sel` and `upd`

```
forall M, A1, A2, V.
    A1 == A2 => sel(upd(M, A1, V), A2) == V

forall M, A1, A2, V.
    A1 /= A2 => sel(upd(M, A1, V), A2) == sel(M, A2)
```

Updated Hoare-Rule for References

New rule for **deref-read**

```
{B [sel(M,y)/x]} x := *y {B}
```

New rule for **deref-write**

```
{B [upd(M,x,e)/M]} *x := e {B}
```

Assignments and Aliasing Revisited

In general, for what `P` is the following valid?

```
{P}    *x = 5    {*x + *y == 10}
```

Or rather,

```
{P}    *x = 5    {sel(M,x) + sel(M,y) == 10}
```

Now, with the new **deref-write** rule `P` becomes

TODO FIX THIS ~~~~~{.javascript} A = [upd(M, x, 5)/M] (x+y=10) = [upd(M, x, 5)/M] (sel(M,x) + sel(M,y) = 10) = sel(upd(M, x, 5), x) + sel(upd(M, x, 5), y) = 10 = 5 + sel(upd(M, x, 5), y) = 10 = sel(upd(M, x, 5), y) = 5 = (x = y & 5 = 5) || (x != y & sel(M, y) = 5) = x=y || *y = 5 ~~~~~

Which is exactly what we wanted!

Deductive Verifiers

We have just scratched the surface

Many *industrial strength* verifiers for real languages

- Why3
- ESC-Java

And these, which you can play with online

- VCC
- Spec# 1 Spec# 2
- Verifast

All very impressive: Try them out and see!

Main hassle: writing invariants, pre and post...